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A Note on Fuzzy Set with Some Basic Concepts and Its Verifications

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***Abstract*—The paper discusses some fundamental fuzzy set notions that are pertinent to the subjects discussed. Furthermore, it introduces fuzzy sets, discourse universes, linguistic variables, linguistic values, membership functions, Features of Membership Functions, operations on fuzzy sets, and verification of various properties of those operations' use of fuzzy sets.**

***Index Terms*— Fuzzy sets, R-functions and L-functions, Features of Membership Functions, Operation on fuzzy set.**

I. Introduction

The concept of uncertainty is one of the many fundamental transformations that have occurred in science and mathematics this century. The steady shift away from the conventional view, which maintains that uncertainty is undesirable in science and should be avoided at all costs, and towards an alternative view, which is tolerant of uncertainty and maintains that science cannot escape it, has been the most obvious manifestation of this transformation in science. Traditional thinking holds that science should strive towards certainty in all of its forms (precision, specificity, sharpness, consistency, etc.); as a result, uncertainty (imprecision, non-specificity, vagueness, inconsistency, etc.) is viewed as unscientific. The opposing, or modern, view holds that uncertainty is not just an inevitable scourge but also has significant benefit. It is even thought to be necessary for science.

It is generally agreed that an important point in the evolution of the modem concept of uncertainty was the publication of a seminal paper by Lotfi A. Zadeh (1965b), even though. Some ideas presented in the paper were envisioned some 30 years earlier by the American philosopher Max Black (1937). In his paper, Zadeh introduced a theory whose objects-fuzzy sets-are sets with boundaries that are not precise. The membership in a fuzzy set is not a matter of affirmation or denial, but rather a matter of a degree. The significance of Zadeh's paper was that it challenged not only probability theory as the sole agent for uncertainty, but the very foundations upon which probability theory is based: Aristotelian two-valued logic. When A is a fuzzy set and x is a relevant object, the proposition "x is a member of A" is not necessarily either true or false, as required by two-valued logic, but it may be true only to some degree, the degree to which x is actually a member of A. It is most common, but not required, to express degrees of membership in fuzzy sets as well as degrees of truth of the associated propositions by numbers in the closed unit interval [0, 1]. The extreme values in this interval, 0 and 1, then represent, respectively.

A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus, individuals may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. As already mentioned, these membership grades are very often represented by real-number values ranging in the closed interval between 0 and 1. Thus, a fuzzy set representing our concept of sunny might assign a degree of membership of 1 to a cloud cover of 0%, .8 to a cloud cover of 20%, .4 to a cloud cover of 30%, and 0 to a cloud cover of 75%. These grades signify the degree to which each percentage of cloud cover approximates our subjective concept of sunny, and the set itself models the semantic flexibility inherent in such a common linguistic term. Because full membership and full non-membership in the fuzzy set can still be indicated by the values of 1 and 0, respectively, we can consider the concept of a crisp set to be a restricted case of the more general concept of a fuzzy set for which only these two grades of membership are allowed.

Research on the theory of fuzzy sets has been growing steadily since the inception of the theory in the mid-1960s. The body of concepts and results pertaining to the theory is now quite impressive. Research on a broad variety of applications has also been very active and has produced results that are perhaps even more impressive. In this manuscript, we present an introduction to the major developments of the theory as well as to some of the most successful applications of the theory

II. Some Elementary Concepts of Fuzzy Logic

A. *Classical set vs. fuzzy set*

Classical set contains elements that satisfy precise properties of membership while fuzzy set contains elements that satisfy imprecise properties of membership. Classical set theory allows the membership of the elements in the set in binary terms.

Fuzzy set theory permits membership function valued in the interval [0,1].

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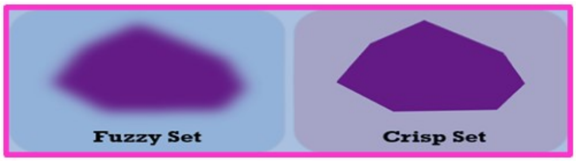

Fig.1. Representation of Fuzzy set and Crisp set.

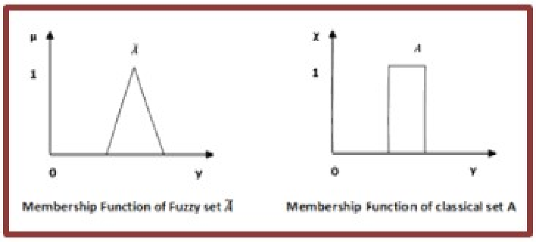

Fig. 2. Membership function of Fuzzy set and classical set

TABLE I: CLASSICAL SET VS. FUZZY SET

Fuzzy set	Crisp set (Classical set)
Indeterminate boundaries	Determinate boundaries
Uncertainty about the location of the set boundaries.	Certainty about the location of the set boundaries
Used in fuzzy controllers.	Used in digital system design.

B. The Universe of Discourse (UoD)

All elements in a set are taken from a universe of discourse or universe set that contains all the elements that can be taken into consideration when the set is formed. In reality there is no such thing as a set or a fuzzy set because all sets are subsets of some universe set, even though the term 'set' is predominantly used. In the fuzzy case, each element in the universe set is a member of the fuzzy set to some degree, even zero. The set of elements that have a non-zero membership is referred to as the support. We will use the notation U for the universe set.

$A = \{(x, \mu_A(x)) \mid x \in U\}$

Fuzzy set **Membership function** **Universe or universe of discourse**

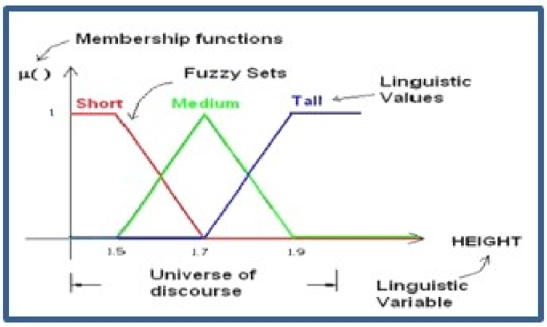

Fig 3.The Fuzzy sets are represented in universe of discourse

C. Mathematical Concept of fuzzy set

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function μ_A: $X \to [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$

It is clear that A is completely determined by the set of tuples $A = \{(u, \mu_A(u)) \mid u \in X\}$. Frequently we will write A(x) instead of $\mu_A(x)$. The family of all fuzzy sets in X is denoted by F(X) . If $X = \{x_1, \cdots, x_n\}$ is a finite set and A is a fuzzy set in X then we often use the notation $A = \mu_1/x_1 + \cdots \mu_n/x_n$ where the term μ_i/x_i , $i = 1, \cdots, n$ signifies that μ_i is the grade of membership of x_i in A and the plus sign represents the union.

D. Representation of fuzzy set

Consider two cases of universe of information and understand how a fuzzy set can be represented.

Case i: When universe of information U is discrete and finite. $A = \{\sum_{i=1}^{n} \frac{\mu_A(x_i)}{x_i}\}$

Case ii:When universe of information U is continuous and infinite. $A = \{\int \mu_{A(x)} / x\}$

III. FEATURES OF MEMBERSHIP FUNCTIONS

A. Support of a Membership Function

Let A be a fuzzy subset of X; the support of A, denoted supp(A), is the crisp subset of X whose elements all have nonzero membership grades in A.

$\mathrm{supp}(A) = \{x \in X \mid A(x) > 0\}$.

B. Core of a Membership Function

Core of a membership function for a fuzzy set A is defined as that region of universe that is characterized by complete or full membership in the set A. Therefore core consists of all those elements X of universe of discourse, such that

$\mathrm{Core}\ (A) = \{x \in X \mid A(x) = 0\}$.

C. Boundary of a Membership Function

Boundary of a membership function for a fuzzy set A is defined as that region of universe X, that is characterized by non-zero membership but not complete membership. Boundaries comprises that part of elements X of Universe of discourse whose membership value is given by

$\mathrm{Boundary}\ (A) = \{x \in X \mid A(x) \in (0,1)\}$.

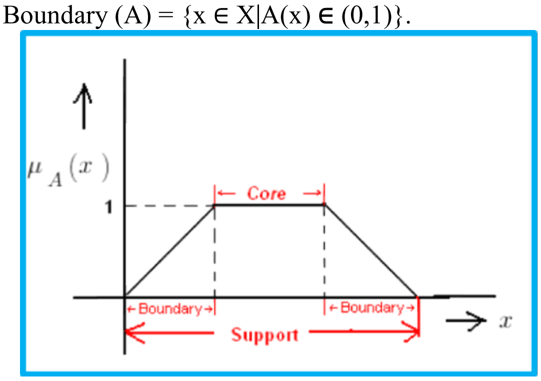

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Fig. 4. Core, support and boundaries of fuzzy membership function

D. Cross-over Points of a Membership Function

It is defined as the elements of a fuzzy set A whose membership value is equal to 0.5.

$\mu_{A(x)} = 0.5$

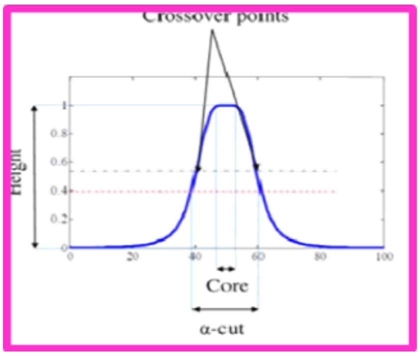


Fig. 5. Core, support and boundaries of fuzzy membership function

E. Height of a Membership Functions

Height of a membership function is the maximum value of the membership function. If the height of a fuzzy set is < 1 then it subnormal fuzzy set. Whereas if its height is equal to 1 then it is a normal fuzzy set.

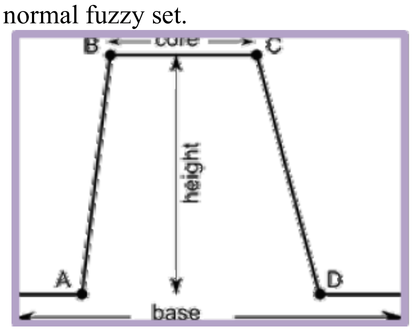


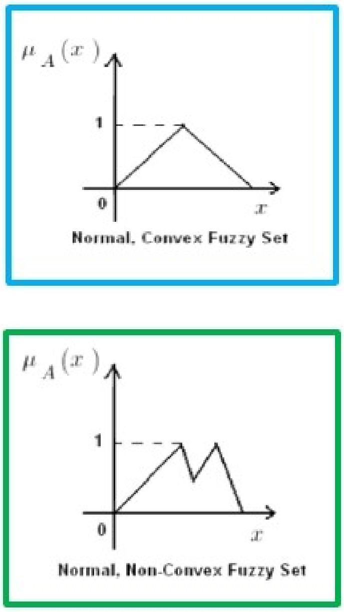
Fig. 6. Height of a Membership Functions of fuzzy set.

F. Normal Fuzzy Set

A normal fuzzy set is one that consists of at-least one element 'x' of universe whose membership value is unity. For fuzzy sets where only one element which has a membership value of unity, that particular element is called prototype of the fuzzy set or prototypical element

G. Convex Fuzzy Set

Convex fuzzy set is described by a membership function whose membership values are strictly Monotonically Increasing or Monotonically Decreasing or Initially Monotonically Increasing then Monotonically Decreasing with the increase in the values of the elements of that particular fuzzy set.



(a)

(b)

Fig. 7. (a) Normal convex Fuzzy set, (b) Non-convex Fuzzy set

H. Fuzzy Number

If 'A' is a convex single point normal fuzzy set defined on real line, then 'A' is called Fuzzy Number

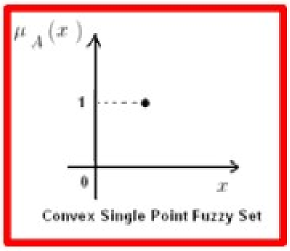


Fig. 8. Convex single point Fuzzy set

I. Fuzzy α-cut

The **α-cut** of a fuzzy set A is a crisp set defined by $A_\alpha = \{ x \mid \mu_{\underline{A}}(x) \geq \alpha \}$

Strong α-cut of a fuzzy set A is a crisp set defined by $A_\alpha + = \{ x \mid \mu_{\underline{A}}(x) > \alpha \}$

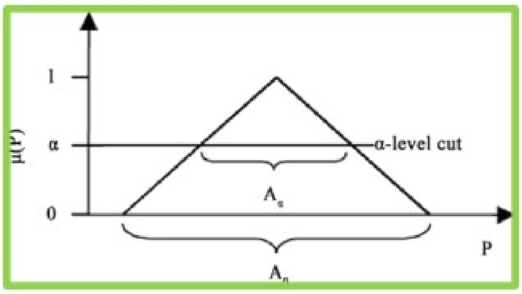


Fig. 9. α-cut of a Fuzzy set

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J. Fuzzy singleton

Fuzzy set whose support is a single point in X with μA (x) =1 is called fuzzy singleton

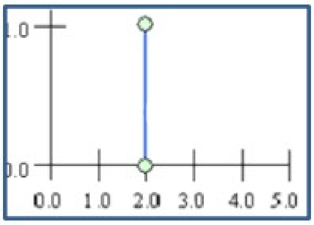

Fig. 10 Singleton of the Fuzzy set

K. Fuzzy subset

If the membership grade of each element of the universal set U in fuzzy set A is less than or equal to its membership grade in fuzzy set B, then A is called a subset of set B, that is, if for every x ∈ U , μA(x) ≤ μB(x) then A ⊆B

IV. Membership Functions

A. Definition

Each element of X is mapped to a value between 0 and 1, and the membership function for a fuzzy set A on the universe of discourse X is defined as μA: X → [0,1]. The degree to which the element in X is a member of the fuzzy set A is quantified by this value, also known as the membership value or degree of membership.

B. Triangular Membership function

$$\mu_A(x) = \begin{cases} 0 & if\ x \le a \\ \frac{x-a}{b-a} & if\ a \le x \le b \\ \frac{c-x}{c-b} & if\ b \le x \le c \\ 0 & if\ x \ge c \end{cases}$$

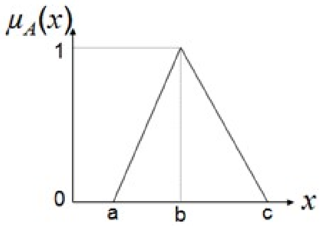

Fig. 11 Triangular Membership functions of the Fuzzy set

Let a, m and b represent the x coordinates of the three vertices of μA(x) in a fuzzy set A ('a' denote the lower boundary and 'b' denote the upper boundary where membership degree is zero, 'm' denote the centre where membership degree is 1).

C. Trapezoidal Membership function

Defined by a lower boundary 'a', an upper boundary 'd', a lower support boundary 'b', and an upper support boundary 'c', where a < b < c < d.

$$\mu_A(x) = \begin{cases} 0, & (x < a) \text{ or } (x > d) \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \end{cases}$$

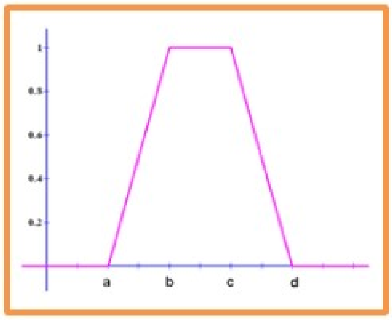

Fig. 12 Trapezoidal Membership functions of the Fuzzy set

There are two special cases of a trapezoidal function, which are called R-functions and L-functions:

(i). (i) R-functions: with parameters a = b = - ∞

$$\mu_A(x) = \begin{cases} 0, & x > d \\ \frac{d-x}{d-c}, & c \le x \le d \\ 1, & x < c \end{cases}$$

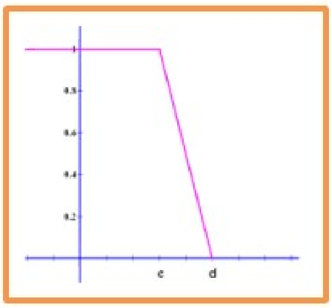

Fig. 13 Trapezoidal Membership R- functions of the Fuzzy set

(ii). L-Functions: with parameters c = d = + ∞

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$$\mu_A(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$$

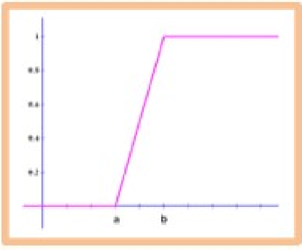

Fig. 14. Trapezoidal Membership L- functions of the Fuzzy set.

D. *Gaussian membership functions*

Defined by a central value 'm 'and a standard deviation k > 0. [2] discussed that Biomedical and anatomical data are made simple to acquire because of progress accomplished in computerizing picture division. More research and work on it has improved more viability to the extent the subject is concerned. A few tech- niques are utilized for therapeutic picture division, for example, Clustering strategies, Thresholding technique, Classifier, Region Growing, Deformable Model, Markov Random Model and so forth. The smaller k is, the narrower the "bell" is.

$$\mu_A(x) = e^{-\frac{(x-m)^2}{2k^2}}$$

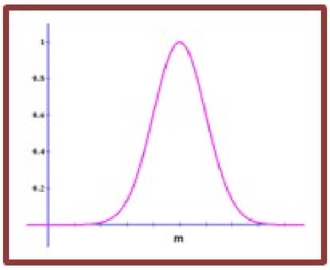

Fig.15. Gaussian membership functions of the Fuzzy set.

E. *Generalized bell shape Membership function*

It is also called Cauchy Membership function. A generalized bell MF is specified by three parameters {a, b, c} and can be defined as follows.

$$f(x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$

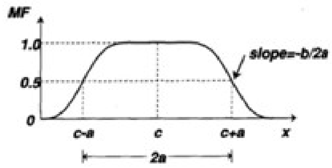

Fig.16. Generalized bell shape Membership function

F. *Sigmoid membership function*

It is represented as follows

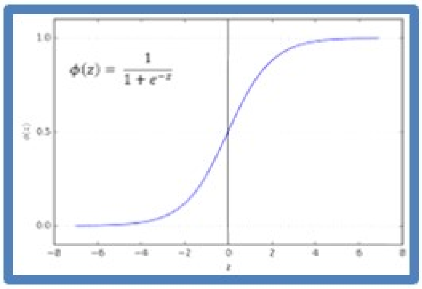

Fig.17. Sigmoid Membership function

V. Operations on Fuzzy Sets

A. *Union of fuzzy set*

Let μ_A and μ_B be membership functions that define the fuzzy sets A and B, respectively, on the universe X. The union of fuzzy sets A and B is a fuzzy set defined by the membership function: $\mu_{A\cup B}(x) = \text{Max}\ (\mu_A(x), \mu_B(x))$

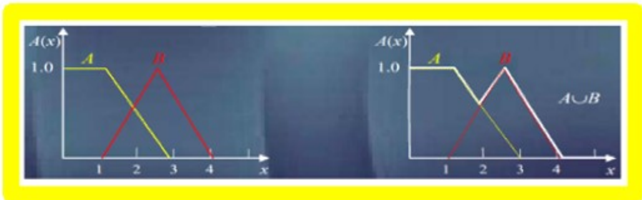

Fig.18. Union Operations on Fuzzy Sets

B. *Intersection of fuzzy set*

Let μ_A and μ_B be membership functions that define the fuzzy sets A and B, respectively, on the universe X. The intersection of fuzzy sets A and B is a fuzzy set defined by the membership function: $\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$

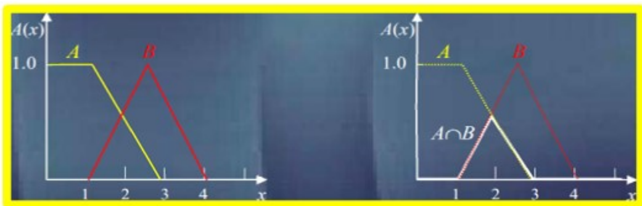

Fig.19. Intersection Operations on Fuzzy Sets

C. *Complement of fuzzy set*

Let μ_A be a membership function that defines the fuzzy set A, on the universe X. The complement of A is a fuzzy set defined by the membership function: $\mu_A{}^c(x) = 1 - \mu_A(x)$

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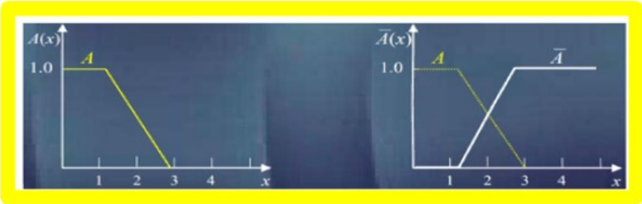

Fig.20. Standard Operations on Fuzzy Sets

VI. Linguistic Variables, Linguistic Values, Linguistic Terms

A. Linguistic Variables

A linguistic variable is a variable whose values are not numbers, but words or sentences in a natural language. For example,

IF room IS cold THEN heat IS on; IF room IS hot THEN heat IS off;

In the above example, room and heat are crisp variables, and hot, cold, on and off are linguistic variables. The linguistic variables on and off in the above example are represented in the crisp variable heat as a 1 and a 0 respectively. The hot and cold linguistic variables represent a range of values corresponding to the crisp variable room. [4] discussed about the combination of Graph cut liver segmentation and Fuzzy with MPSO tumor segmentation algorithms. The system determines the elapsed time for the segmentation process. The accuracy of the proposed system is higher than the existing system. The algorithm has been successfully tested in multiple images where it has performed very well, resulting in good segmentation. It has taken high computation time for the graph cut processing algorithm. In future work, we can reduce the computation time and improves segmentation accuracy. This relationship can be represented as shown in the following graph.

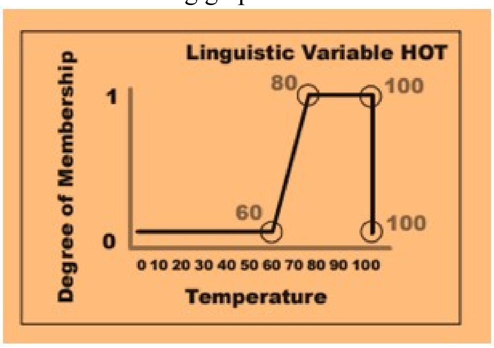

Fig. 21. Linguistic variables

B. Linguistic values and linguistic terms:

A

Notations	Stands for	Linguistic Values
NL	Negative large	Very Low
NS	Negative small	Low
ZE	Zero	Normal
PS	Positive small	High
PL	Positive large	Very High

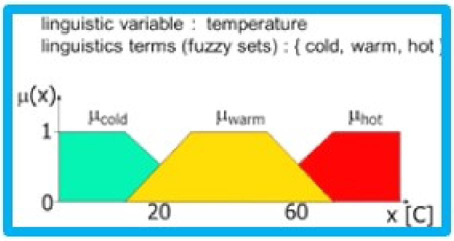

Fig. 22. Linguistic values and linguistic terms

VII. Some Properties of the Operations with Fuzzy Sets:

Let F, G, H are fuzzy matrix. Then

1. Commutativity:
i)F ∪ G = G ∪ F
ii)F ∩ G = G ∩ F
2. Associativity:
i) (F ∪ G) ∪ H = F ∪ (G ∪ H)
ii) (F ∩ G) ∩ H = F ∩ (G ∩ H)
3. Distributivity:
i) F ∪ (G ∩ H) = (F ∪ G) ∩ (F ∪ H)
ii) F ∩ (G ∪ H) = (F ∩ G) ∪ (F ∩ H)
4. Idempotency:
i) F ∪ F = F
ii) F ∩ F = F
5. Identity:
i) F ∪ Ø = Ø ∪ F = F
ii) F ∩ Ø = Ø ∩ F = Ø

Verification of some properties of the operations with fuzzy sets: Example

1. Commutativity:
i) F∪G = G∪F
(∪-maximum)

$$F = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.7 & 0.8 & 0.9 \\ 0.6 & 0.5 & 0.4 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.1 & 0.9 & 0.8 \\ 0.7 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.3 \end{pmatrix}$$

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$$F \cup G = \begin{pmatrix} 0.1 & 0.9 & 0.8 \\ 0.7 & 0.8 & 0.9 \\ 0.6 & 0.5 & 0.4 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.1 & 0.9 & 0.8 \\ 0.7 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.3 \end{pmatrix}$$

$$F = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.7 & 0.8 & 0.9 \\ 0.6 & 0.5 & 0.4 \end{pmatrix}$$

$$G \cup F = \begin{pmatrix} 0.1 & 0.9 & 0.8 \\ 0.7 & 0.8 & 0.9 \\ 0.6 & 0.5 & 0.4 \end{pmatrix}$$

Hence verified F∪G = G∪F

ii) F∩G = G∩F
(∩-minimum)

$$F = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.7 & 0.8 & 0.9 \\ 0.6 & 0.5 & 0.4 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.1 & 0.9 & 0.8 \\ 0.7 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.3 \end{pmatrix}$$

$$F \cap G = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.7 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.3 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.1 & 0.9 & 0.8 \\ 0.7 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.3 \end{pmatrix}$$

$$F = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.7 & 0.8 & 0.9 \\ 0.6 & 0.5 & 0.4 \end{pmatrix}$$

$$G \cap F = \begin{pmatrix} 0.1 & 0.2 & 0.3 \\ 0.7 & 0.6 & 0.5 \\ 0.4 & 0.2 & 0.3 \end{pmatrix}$$

2. Associativity: i) (F∪G) ∪H = F∪ (G∪H)

$$F = \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.5 & 0.6 \\ 0.6 & 0.7 \end{pmatrix}$$

$$H = \begin{pmatrix} 0.9 & 0.8 \\ 1 & 0.2 \end{pmatrix}$$

$$F \cup G = \begin{pmatrix} 0.5 & 0.6 \\ 0.6 & 0.7 \end{pmatrix}$$

$$(F \cup G) \cup H = \begin{pmatrix} 0.9 & 0.8 \\ 1 & 0.7 \end{pmatrix}$$

F∪ (G∪H):

$$F = \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.5 & 0.6 \\ 0.6 & 0.7 \end{pmatrix}$$

$$H = \begin{pmatrix} 0.9 & 0.8 \\ 1 & 0.2 \end{pmatrix}$$

$$G \cup H = \begin{pmatrix} 0.9 & 0.8 \\ 1 & 0.7 \end{pmatrix}$$

$$F \cup (G \cup H) = \begin{pmatrix} 0.9 & 0.8 \\ 1 & 0.7 \end{pmatrix}$$

ii) (F ∩ G) ∩ H = F ∩ (G ∩ H)

$$F = \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{pmatrix}$$

$$G = \begin{pmatrix} 0.5 & 0.6 \\ 0.6 & 0.7 \end{pmatrix}$$

$$H = \begin{pmatrix} 0.9 & 0.8 \\ 1 & 0.2 \end{pmatrix}$$

$$F \cap G = \begin{pmatrix} 0.1 & 0.2 \\ 0.3 & 0.4 \end{pmatrix}$$

$$(F \cap G) \cap H = \begin{pmatrix} 0.9 & 0.8 \\ 1 & 0.7 \end{pmatrix}$$

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F ∩ (G ∩ H):

$$F=\begin{pmatrix}0.1 & 0.2\\0.3 & 0.4\end{pmatrix}$$

$$G=\begin{pmatrix}0.5 & 0.6\\0.6 & 0.7\end{pmatrix}$$

$$H=\begin{pmatrix}0.9 & 0.8\\1 & 0.2\end{pmatrix}$$

$$G\cap H=\begin{pmatrix}0.5 & 0.6\\0.6 & 0.2\end{pmatrix}$$

$$\mathrm{F}\cap(\mathrm{G}\cap H)=\begin{pmatrix}0.9 & 0.8\\1 & 0.7\end{pmatrix}$$

Hence verified (F ∩ G) ∩ H = F ∩ (G ∩ H)

Hence verified (F ∩ G) ∩ H = F ∩ (G ∩ H)

3. Distributivity:
i) FU (G ∩ H) = (FUG) ∩ (FUH) ii) F ∩ (GUH) = (F ∩ G) ∪ (F ∩ H)

$$\mathrm{F}=\begin{vmatrix}1 & 0.2 & 0.6 & 0.6\\0.8 & 0 & 0.8 & 0.4\\0.6 & 0.2 & 1 & 0.2\\0.4 & 0.4 & 0.8 & 0\end{vmatrix}$$

$$G=\begin{vmatrix}0 & 1 & 0.2 & 1\\0.2 & 0.8 & 0.5 & 0.4\\0.6 & 0.4 & 0.7 & 0.3\\0.8 & 0.2 & 0.9 & 0.2\end{vmatrix}$$

$$H=\begin{vmatrix}0.2 & 0.6 & 1 & 0.4\\0.3 & 0.7 & 0.1 & 0\\0.4 & 0.8 & 0.2 & 0.5\\0.5 & 0.9 & 0.3 & 0.6\end{vmatrix}$$

$$\mathrm{G}\cap\mathrm{H}=\begin{vmatrix}0 & 0.6 & 0.2 & 0.4\\0.2 & 0.7 & 0.1 & 0\\0.4 & 0.4 & 0.2 & 0.3\\0.5 & 0.2 & 0.3 & 0.2\end{vmatrix}$$

$$\mathrm{F}\cup(\mathbf{G}\cap\mathbf{H})=\begin{vmatrix}1 & 0.6 & 0.6 & 0.6\\0.8 & 0.7 & 0.8 & 0.4\\0.6 & 0.4 & 1 & 0.3\\0.5 & 0.4 & 0.8 & 0.2\end{vmatrix}$$

$$\mathrm{F}\cup\mathrm{G}=\begin{vmatrix}1 & 1 & 0.6 & 1\\0.8 & 0.8 & 0.8 & 0.4\\0.6 & 0.4 & 1 & 0.3\\0.8 & 0.4 & 0.9 & 0.2\end{vmatrix}$$

$$\mathrm{F}\cup H=\begin{vmatrix}1 & 0.6 & 1 & 0.6\\0.8 & 0.7 & 0.8 & 0.4\\0.6 & 0.8 & 1 & 0.5\\0.5 & 0.9 & 0.8 & 0.6\end{vmatrix}$$

$$(\mathrm{F}\cup\mathbf{G})\cap(\mathrm{F}\cup\mathbf{H}=\begin{vmatrix}1 & 0.6 & 0.6 & 0.6\\0.8 & 0.7 & 0.8 & 0.4\\0.6 & 0.4 & 1 & 0.3\\0.5 & 0.4 & 0.8 & 0.2\end{vmatrix}$$

Hence verified FU (G ∩ H) = (FUG) ∩ (FUH)

ii) F ∩ (GUH) = (F ∩ G) ∪ (F ∩ H)

$$\mathrm{F}=\begin{vmatrix}1 & 0.2 & 0.6 & 0.6\\0.8 & 0 & 0.8 & 0.4\\0.6 & 0.2 & 1 & 0.2\\0.4 & 0.4 & 0.8 & 0\end{vmatrix}$$

$$G=\begin{vmatrix}0 & 1 & 0.2 & 1\\0.2 & 0.8 & 0.5 & 0.4\\0.6 & 0.4 & 0.7 & 0.3\\0.8 & 0.2 & 0.9 & 0.2\end{vmatrix}$$

$$H=\begin{vmatrix}0.2 & 0.6 & 1 & 0.4\\0.3 & 0.7 & 0.1 & 0\\0.4 & 0.8 & 0.2 & 0.5\\0.5 & 0.9 & 0.3 & 0.6\end{vmatrix}$$

$$\mathrm{G}\cup\mathrm{H}=\begin{vmatrix}0.2 & 1 & 1 & 1\\0.3 & 0.8 & 0.5 & 0.4\\0.6 & 0.8 & 0.7 & 0.5\\0.8 & 0.9 & 0.9 & 0.6\end{vmatrix}$$

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$$F\cap(\mathbf{G}\cup\mathbf{H})=\begin{vmatrix}0.2&0.2&0.6&0.6\\0.3&0&0.5&0.4\\0.6&0.2&0.7&0.2\\0.4&0.4&0.8&0\end{vmatrix}$$

$$F\cap G=\begin{vmatrix}0&0.2&0.2&0.6\\0.2&0&0.5&0.4\\0.6&0.2&0.7&0.2\\0.4&0.2&0.8&0\end{vmatrix}$$

$$F\cap H=\begin{vmatrix}0.2&0.2&0.6&0.4\\0.3&0&0.1&0\\0.4&0.2&0.2&0.2\\0.4&0.4&0.3&0\end{vmatrix}$$

$$(F\cap\mathbf{G})\cup(F\cap\mathbf{H})=\begin{vmatrix}0.2&0.2&0.6&0.6\\0.3&0&0.5&0.4\\0.6&0.2&0.7&0.2\\0.4&0.4&0.8&0\end{vmatrix}$$

Hence verified F ∩ (G∪H) = (F ∩ G) ∪ (F ∩ H)

4. Idempotency:
i) F∪F = F
ii) F ∩ F = F

$$F=\begin{pmatrix}0.1&0.2\\0.3&0.4\end{pmatrix}$$

$$F\cup F=\begin{pmatrix}0.1&0.2\\0.3&0.4\end{pmatrix}=F$$

$$F\cap F=\begin{pmatrix}0.1&0.2\\0.3&0.4\end{pmatrix}=F$$

Hence verified i) F∪F = Fii) F ∩ F = F

5. Identity:
F∪∅ = ∅∪F = FG) F ∩ ∅ = ∅ ∩ F = ∅L

$$F=\begin{pmatrix}0.6&0.8\\0.3&0.4\end{pmatrix}$$

$$\varnothing=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$F\cup\varnothing=\varnothing\cup F=\begin{pmatrix}0.6&0.8\\0.3&0.4\end{pmatrix}=F$$

$$F\cap\varnothing=\varnothing\cap F=\begin{pmatrix}0&0\\0&0\end{pmatrix}=\varnothing$$

Hence verified that F∪∅ = ∅∪F = Fii) F ∩ ∅ = ∅ ∩ F = ∅

VIII. Conclusions

This paper introduces some fundamental ideas, fundamental fuzzy set operations, and the use of fuzzy set theory for the testing of some basic operation attributes. According to theory, fuzzy expert systems could be used in any human endeavor where knowledge, experience, and approximate reasoning are prevalent.

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