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# Exploration of the Application of Vectors in Elementary Algebra

Yifan Li, Liang Fang*, Rui Chen

College of Mathematics and Statistics, Taishan University, Tai'an, China

Email:fangliang3@163.com

***Abstract*—As an important concept in elementary mathematics, vector is an important tool to connect algebra and geometry, and it is also one of the compulsory contents of the national college entrance examination. This article mainly explores the application of vectors in proving inequalities and identities in elementary algebra, and solving the extremum problem using vectors.**

***Index Terms*—Vector, Elementary algebra, Standard frame, Cauchy Schwartz inequality, coordinate.**

## I. Introduction

In multiple disciplines such as mechanics, physics, and engineering technology, we have already understood the basic concept of vectors. In these disciplines, vectors are endowed with many practical meanings, such as displacement, velocity, acceleration, etc. In elementary mathematics, we break away from these practical meanings and use a directed line segment to represent vectors more vividly.

At first, we recognized forces and accelerations in physics, which was the initial image of vectors. As early as 350 BC, Aristotle discussed mechanical problems in his works and expressed forces as vectors. However, the earliest person who abandon the practical significance of vectors and use directed line segments to represent vectors was the British scientist Newton [1-2].

In the late 18th century, Norwegian surveyor Wessel first associated complex numbers with coordinates and used complex number operations to define vector operations, linking complex numbers with geometry [3]. However, the use of complex numbers was limited, as it only remained in the representation of planes. In the mid-19th century, Irish mathematician Hamilton invented quaternions and extended plane vectors to geometric spaces [4].

At this time, people have slowly accepted vector, but vector still does not occupy a unique position in mathematics. Until the 1880s, Jubers and Heiveside of Britain introduced the quantity product and vector product of vector, and they believed that a vector is only the vector part of quaternion [5-6]. Since then, people have more widely used vector in mathematical analysis and analytic geometry, and vector algebra has been greatly developed.

At present, many algebraic and geometric problems in elementary mathematics are difficult to prove or calculate, too complex or highly skilled, and some problems even need to add more auxiliary lines, which makes it very difficult for us to solve problems. Therefore, compared with using formula theorem to solve problems, vector method will be much simpler. We introduce vectors in elementary algebra to transform these problems into vector operations, To deal with these problems from another point of view, how to better use vectors to solve problems in elementary mathematics has become an important topic.

## II. Concepts Related to Vectors

Before we learned about vectors in elementary mathematics, we had been exposed to such quantities as work, density, time, volume, etc. in physics and daily life. These quantities can be expressed by a number in its own unit. This kind of quantity with only size is called a number or scalar quantity. It can be inferred that other quantities such as displacement, speed, acceleration, etc., which are relative to quantity, have direction as well as size. This quantity is called a vector.

**Definition 2.1** A quantity that has both size and direction is called a vector, or vector, abbreviated as a vector [7].

In geometry, we consider a vector as a directed line segment with an arrow. The starting and ending points of this line segment are the starting and ending points of the vector. The direction pointed by the arrow on a directed line segment is the direction of the vector, and the size of the vector is represented by the length of the directed line segment.

**Definition 2.2** The size of a vector is called the modulus of the vector, also known as the length of the vector.

Therefore, we stipulate that the vector whose module is equal to 1 is called unit vector; A vector with a modulus equal to 0 is called a zero vector.

**Definition 2.3** If the modules of two vectors are equal and the directions are the same, then these two vectors are called equal vectors. All zero vectors are equal.

**Definition 2.4** A fixed point $O$ in space, along with all three non coplanar ordered vectors $\vec{e_1}$, $\vec{e_2}$, $\vec{e_3}$, is called a frame in geometric space, denoted as $\{O;\vec{e_1},\vec{e_2},\vec{e_3}\}$.

**Definition 2.5** For any point $P$ in the space where the reference frame $\{O;\vec{e_1},\vec{e_2},\vec{e_3}\}$ is taken, the vector $\overrightarrow{OP}$

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is called the vector of point $P$, or the position vector of point $P$. If $\overrightarrow{OP} = x\vec{e}_1 + y\vec{e}_2 + z\vec{e}_3$, , then $\{x, y, z\}$ is called the coordinate of vector $\overrightarrow{OP}$ with respect to reference frame $\{O;\vec{e}_1,\vec{e}_2,\vec{e}_3\}$. At this point, the coordinate of point $P$ with respect to reference frame $\{O;\vec{e}_1,\vec{e}_2,\vec{e}_3\}$ is denoted as $P(x, y, z)$ or $(x, y, z)$. [11] examined the development and refinement of possible mathematical models for the intellectual system of career guidance. Mathematical modeling of knowledge expression in the career guidance system, Combined method of eliminating uncertainties, Chris-Naylor method in the expert information system of career guidance, Shortliff and Buchanan model in the expert information system of career guidance and DempsterSchafer in the expert information system of career guidance method has been studied. The algorithms of the above methods have been developed. The set of hypotheses in the expert system is the basic structure of the system that determines the set of possible decisions of the expert system. This set, which is crucial in decision-making, should be sufficiently complete to describe all the possible consequences of situations that arise in the subject area. Therefore, it is important to improve the mathematical models of the intellectual system of career guidance. [12] discussed about specific Policy document which ensures of which the teaching, learning in addition to assessment methods are upwards to the amount of typically the course and are ideal to the attainment involving objectives and intended understanding outcomes of the program and the course. The particular policy requires that school members use recent in addition to variety of teaching, mastering methods and assessment methods. Higher Quality Accredited Institutions will continue to further more improve the standard involving teaching and learning via recognition, sharing and moving of good practices to be able to inspire the learners to be able to achieve their potentials throughout a multicultural environment in addition to in turn, improve accomplishment, retention and learners pleasure

## III. The Application of Vectors in Elementary Algebra

### *A. Using Vector to Prove Inequalities*

In elementary algebra, we often encounter inequality proof problems. If we directly solve them using algebraic methods, many formulas and properties will be used, and the calculation process is more complex and cumbersome. In this case, vector methods can be used to solve these problems, which will be more concise.

**Example 1**. Using the product of two vectors to prove the Cauchy-Schwarz Inequality

$$\left(\sum_{i=1}^{3} a_i b_i\right)^2 \le \sum_{i=1}^{3} a_i^{\,2} \sum_{i-1}^{3} b_i^{\,2}.$$

**Proof**. Assuming $\vec{a} = \{a_1, a_2, a_3\}$, $\vec{b} = \{b_1, b_2, b_3\}$, then $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\angle(\vec{a},\vec{b})$.

Due to $-1 \le \cos\angle(\vec{a},\vec{b}) \le 1$, we have

$$|\vec{a}\cdot\vec{b}| \le |\vec{a}|\cdot|\vec{b}|,$$

i.e., $\left|\sum_{i=1}^{3} a_i b_i\right| \le \sqrt{\sum_{i=1}^{3} a_i^2}\sqrt{\sum_{i=1}^{3} b_i^2}$ .

Therefore, $\left(\sum_{i=1}^{3} a_i b_i\right)^2 \le \sum_{i=1}^{3} a_i^2 \sum_{i=1}^{3} b_i^2$ .

**Example 2**. Given the condition $a^2 + b^2 + c^2 = 1$, verify that $ab + bc + ca \le 1$.

**Proof**. We construction vectors

$$\vec{m} = \{a, b, c\}, \vec{n} = \{b, c, a\},$$

then $\vec{m}\cdot\vec{n} = ab + bc + ca = |\vec{m}||\vec{n}|\cos\angle(\vec{m},\vec{n})$.

Since

$$|\vec{m}||\vec{n}|\cos\angle(\vec{m},\vec{n}) \le |\vec{m}|\cdot|\vec{n}|$$

$$= \left(\sqrt{a^2 + b^2 + c^2}\right)^2 = 1,$$

so we have $ab + bc + ca \le 1$.

**Example 3**. Given that $x_1, x_2, \cdots, x_n$ are all positive real numbers and satisfy $x_1 + x_2 + \cdots + x_n = a$, it is proven that $\frac{x_1^2}{x_n} + \frac{x_2^2}{x_1} + \cdots + \frac{x_n^2}{x_{n-1}} \ge a$.

**Proof**. Based on the conditions of the question, we construct the vectors

$$\vec{u} = \left(\sqrt{x_n}, \sqrt{x_1}, \cdots, \sqrt{x_{n-1}}\right),$$

$$\vec{v} = \left(\frac{x_1}{\sqrt{x_n}}, \frac{x_2}{\sqrt{x_1}}, \cdots, \frac{x_n}{\sqrt{x_{n-1}}}\right),,$$

then we have

$$|\vec{u}|^2 = x_1 + x_2 + \cdots + x_n = a,$$

$$|\vec{v}|^2 = \frac{x_1^2}{x_n} + \frac{x_2^2}{x_1} + \cdots + \frac{x_n^2}{x_{n-1}},$$

$$|\vec{u}\vec{v}|^2 = (x_1 + x_2 + \cdots + x_n)^2 = a^2.$$

Since $|\vec{u}\vec{v}|^2 \le |\vec{u}|^2|\vec{v}|^2$, so we get

$$a^2 \le a\left(\frac{x_1^2}{x_n} + \frac{x_2^2}{x_1} + \cdots + \frac{x_n^2}{x_{n-1}}\right),$$

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thus $\frac{x_1^2}{x_n}+\frac{x_2^2}{x_1}+\cdots+\frac{x_n^2}{x_{n-1}}\geq a$.

We can see from the above example that when solving such problems, the most crucial thing is to construct vectors that meet the conditions according to the meaning of the problem, and then easily transform inequality problems in algebra into vector problems. By using the special properties of vectors, we can quickly prove them.

***B. Using vectors to prove identities***

For identity problems, we can also prove them by constructing vectors.

**Example 4**. Known $x\sqrt{1-y^2}+y\sqrt{1-x^2}=1$, verify $x^2+y^2=1$.

**Proof**. Let $x=\sin\alpha$, $y=\sin\beta$, then $\sqrt{1-x^2}=\cos\alpha, \sqrt{1-y^2}=\cos\beta$.

So, the equation in the known condition is transformed into $\sin\alpha\cos\beta+\sin\beta\cos\alpha=1$.

Construct vectors

$\vec{m}=\{\sin\alpha,\cos\alpha\}$, $\vec{n}=\{\cos\beta,\sin\beta\}$,

then

$$\vec{m}\cdot\vec{n}=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$=|\vec{m}||\vec{n}|\cos\angle(\vec{m},\vec{n})=\cos\angle(\vec{m},\vec{n})=1.$$

So we get $\vec{m}=\vec{n}$, so $\sin\alpha=\cos\beta$, $\cos\alpha=\sin\beta$.

Therefore,

$$\sin^2\alpha+\sin^2\beta=\sin^2\alpha+\cos^2\alpha=1.$$

Thus, $x^2+y^2=1$ can be obtained.

**Example 5**. Verify that

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

**Proof**. Construct vectors $\vec{m}=\{\cos\alpha,\sin\alpha\}$, $\vec{n}=\{\cos\beta,\sin\beta\}$, then

$$\vec{m}\cdot\vec{n}=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$=|\vec{m}|\cdot|\vec{n}|\cos(\alpha-\beta)=\cos(\alpha-\beta).$$

So $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

**Example 6.** Given

$(a^2+b^2)(c^2+d^2)=(ac+bd)^2 \quad (cd\neq 0)$

verify $\frac{a}{c}=\frac{b}{d}$.

**Proof**. Construct vectors $\vec{m}=\{a,b\}$, $\vec{n}=\{c,d\}$.

If $a=b=0$, the conclusion is clearly valid;

If $a$, $b$ are not all 0, then

$$\cos\angle(\vec{m},\vec{n})=\frac{\vec{m}\cdot\vec{n}}{|\vec{m}|\cdot|\vec{n}|}$$
$$=\frac{ac+bd}{\sqrt{a^2+b^2}\cdot\sqrt{c^2+d^2}}=\pm 1.$$

So $\vec{m}//\vec{n}$. According to the necessary and sufficient conditions for vector collinearity, $\frac{a}{c}=\frac{b}{d}$.

**C. Using Vector to Solve the Maximal Problem**

When it comes to the problem of finding the maximum value of complex irrational functions in algebra, directly using quantity relations or extreme value theory of functions often results in complex calculations, difficult to obtain, and prone to errors. In this case, we can use the inequality property $\left||\vec{a}|-|\vec{b}|\right|\leq|\vec{a}\pm\vec{b}|\leq|\vec{a}|+|\vec{b}|$ to construct vectors appropriately and transform the maximum value problem into a vector problem, cleverly solving the problem.

**Example 7.** Find the minimum value of function $y=\sqrt{x^2+2x+5}+\sqrt{x^2-4x+13}$.

**Solution.** The original function becomes $y=\sqrt{(1+x)^2+2^2}+\sqrt{(2-x)^2+3^2}$.

If the vectors $\vec{m}=\{1+x,2\}$, $\vec{n}=\{2-x,3\}$, are constructed from this, then

$$y=|\vec{m}|+|\vec{n}|\geq|\vec{m}+\vec{n}|=\sqrt{34}.$$

So we get $y_{\min}=\sqrt{34}$.

**Example 8.** Finding the value range of function $y=\sqrt{x^2-2x+5}-\sqrt{x^2+2x+5}$.

Solution. The original formula is

$$y=\sqrt{(x-1)^2+2^2}-\sqrt{(x+1)^2+2^2}.$$

Construct vectors $\vec{a}=\{x-1,2\}$, $\vec{b}=\{x+1,2\}$.

Because $\vec{a}$ and $\vec{b}$ are not collinear, so

$$|y|=\left||\vec{a}|-|\vec{b}|\right|<|\vec{a}-\vec{b}|=2.$$

Therefore, the value range of the original function is $[-2,2]$.

Due to the unique form of the coordinate formula for the product of vector quantities, we can also solve other extremum problems similar to this formula by constructing vectors to simplify complexity.

**Example 9.** Given $x^2+y^2=9$, $a^2+b^2=16$ $(x,y,a,b\in R)$, find the maximum value of $ax+by$.

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Solution. Construct vectors $\vec{m}=\{x,y\}$ and $\vec{n}=\{a,b\}$, then

$$\begin{aligned} ax+by &= \vec{m}\cdot\vec{n} \\ &= |\vec{m}|\cdot|\vec{n}|\cos\angle(\vec{m},\vec{n}) \\ &= 12\cos\angle(\vec{m},\vec{n}). \end{aligned}$$

Because $-12\le 12\cos\angle(\vec{m},\vec{n})\le 12$, so we have

$$-12\le ax+by\le 12.$$

Thus, $(ax+by)_{\max}=12,\ (ax+by)_{\min}=-12.$

## IV. CONCLUSION

As an important bridge connecting algebra and geometry, vector has been running through all branches of mathematics. The basic idea of introducing vector into elementary mathematics and using vector algebra to solve problems is to use the special properties of vector, which has both algebraic form and geometric form, to combine quantitative relations and spatial forms, to quantify and algebraize spatial structures, and to transform abstract qualitative problems into simpler quantitative problems.

In this article, we aim to solve some problems in elementary algebra by utilizing the inequality properties of vectors and special algorithms to construct vectors, simplify complexity, and transform algebraic problems into vector problems for fast problem-solving

Through the full text, we can understand the rich connotation of vector algebra and its wide and profound role in elementary mathematics. The method of vector algebra to solve problems in elementary mathematics can generally be divided into the following steps: (1) find out the relationship between elementary algebra, geometry and vector, such as establishing a coordinate system; (2) Construct vectors or use vectors to represent the elements given in the question; (3) Using the unique properties and special rules of vectors to solve problems; (4) Transform the results of vector operations into corresponding algebraic and geometric relationships

Since vector has the dual identity of algebraic form and geometric form at the same time, it has become a powerful tool for the transformation between algebra and geometry. It can help us avoid complicated calculations and abstract space imagination, and provides us with a new idea to solve problems more concisely. It has become a trend to better use vector methods to solve related problems in elementary mathematics.

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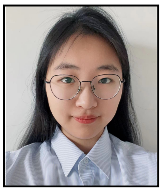
**Authors’ biography with Photos**



**Lifang Li** was born in Mengyin County, Linyi City in December 2002, is a student of Grade 2020 majoring in Mathematics and applied mathematics (normal) in the College of Mathematics and Statistics of Mount Taishan University (student number: 2020061038).

**Liang Fang** was born in December 1970 in Feixian County, Linyi City, Shandong province, China. He is a professor at Taishan University. He obtained his PhD from Shanghai Jiaotong University in June, 2010. His research interests are in the areas of cone optimizations, numerical analysis, and complementarity problems.

**Rui Chen** is a associate professor at Taishan University. She obtained her master's degree from Shandong University in December, 2009. Her research interests are in the areas of application of probability theory, and applied statistics in recent years. email id: chenruimengting@163.com.

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