

# Designing an adaptive controller for a quadrotor in the discrete domain

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**Abstract** - This paper presents the development of an indirect adaptive controller for a Quadrotor in the discrete domain. To assess the superiority of the adaptive controller, the article compares its output response with that of the PI controller. Both controllers demonstrate the capability to maintain system stability, but the adaptive controller achieves significantly lower output errors compared to the PI controller. This highlights the superiority of the adaptive controller, even when system parameters change randomly.

**Index Terms** - Self-Tuning Regulator, adaptive controller, model reference adaptive control, quadrotor.

## I. INTRODUCTION

In the context of the continuous development of science and technology, Quadrotors, alongside other drones, have become popular and versatile in their applications, ranging from industry to research and entertainment. While traditional control methods, such as PID, have found widespread use, they often operate effectively only under specific conditions and cannot adapt to variations in parameters or the working environment. A progressive solution is the utilization of adaptive control methods. This research focuses on developing an adaptive controller for Quadrotors in a discrete domain. This adaptive controller will be designed to automatically adjust to changes in the environment and system parameters, ensuring stable and effective control of the Quadrotor in a variety of situations. This article will detail the design of this adaptive controller and will include simulations using Matlab-Simulink software to evaluate its effectiveness in Quadrotor control. This research holds the promise of serving as a foundation for improving the stability and control quality of Quadrotors in diverse and dynamic environments.

## II. THEORETICAL BASIS

### A. Self-Tuning Regulator (STR)

A self-tuning control system, often abbreviated as STR, is a system designed with the primary objective of performing two simultaneous and vital tasks online:

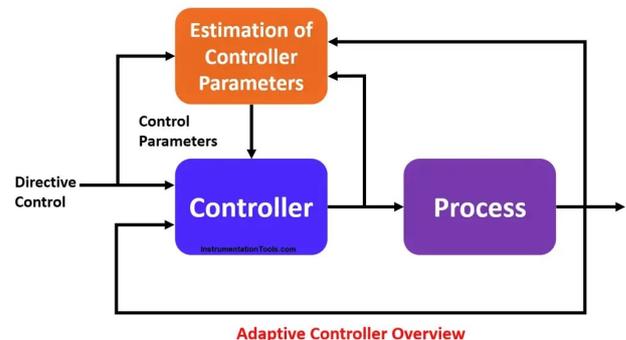


Fig. 1. General block diagram of the self-tuning adaptive control system

- **Model Parameter Identification:** This involves determining the kinematic model parameters of the control object. STR accomplishes this by collecting data from the system and then automatically identifying and updating these parameters.

- **Control Law Calculation:** Building upon the identified model parameters, STR automatically calculates and fine-tunes the control law to ensure optimal control quality for the system.

Self-tuning adaptive control systems can be categorized into two principal types:

- **Indirect Self-Tuning Control System:** In this configuration, the system conducts online estimation of the model parameters of the controlled object and subsequently uses these parameters to compute the controller's parameters.

- Direct Self-Tuning Control System: In this setup, the system directly estimates the controller's parameters without the necessity of identifying the mathematical model of the control object.

This paper will introduce the application of an indirect self-tuning adaptive control system for Quadrotor control.

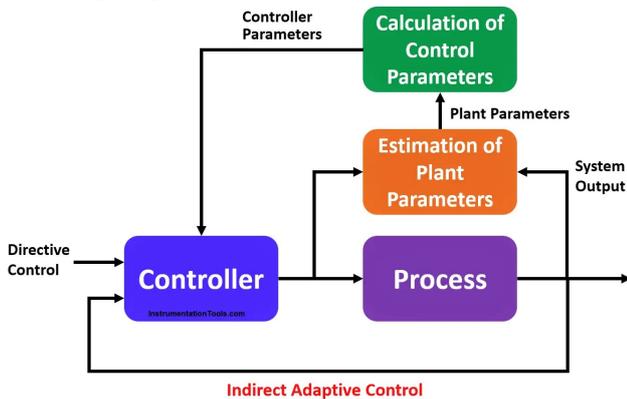


Fig.2. Indirect self-tuning adaptive controller block diagram

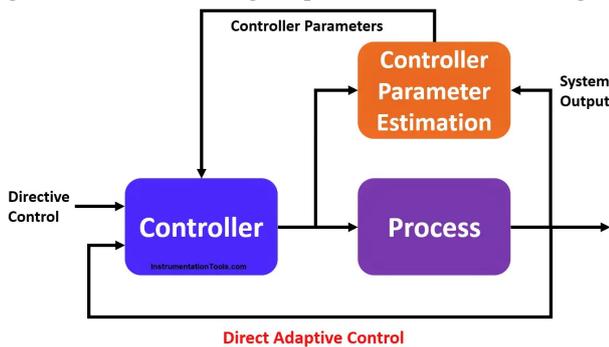


Fig.3. Direct self-tuning adaptive controller block diagram

### B. Sequential Design of an Indirect Self-Tuning Adaptive Control System

The sequential design process for an indirect self-tuning adaptive control system encompasses the following steps:

- *Step 1:* Formulate recursive least squares estimation equations to enable the real-time update of object parameters, whether they represent the transfer function or state equations.

- *Step 2:* Develop expressions that determine the control law as a function of the updated object parameters from Step 1. This includes various control methods such as PID control, standard model control, pole placement control, and LQR control.

- *Step 3:* Conduct simulation experiments to determine the optimal forgetting factor for the recursive least squares estimation algorithm. Adjusting this factor appropriately contributes to the effectiveness and stability of the adaptive control system.

### III. SYNTHESIZING AN INDIRECT SELF-TUNING ADAPTIVE CONTROLLER ACCORDING TO THE SAMPLE MODEL FOR THE QUADROTOR IN THE DISCRETE DOMAIN

#### A. Set the problem

Let the control object be the pitch channel of the quadrotor [1], with a transfer function in the Laplace domain of  $H(p)$ .

$$H(p) = \frac{0,2655 \cdot 14,8528^2}{p^2 + 14,195p + 14,8528^2} = \frac{58.57}{p^2 + 14,2p + 220,6} \quad (1)$$

Analyze unwanted changes in parameters in the transfer function. Synthesis of a self-tuning adaptive controller combining STR-MRAC for quadrotor in the discrete domain so that the response of the closed system follows the standard model  $G_m(p)$ .

$$G_m(p) = \frac{4}{p^2 + 4p + 4} \quad (2)$$

#### B. Analyze the problem

The control object is a combination of the electronic speed controller (ESC), motor, and propeller. In a quadrotor, a brushless DC motor is used. The transfer function of this motor includes a proportional block and a second-order inertial block with two poles related to the mechanical and electrical elements of the motor, respectively. Because the electrical factor has an insignificant influence compared to the influence of the motor's moment of inertia, one pole can be eliminated, and the motor's transfer function can be approximated as a first-order inertial stitch. Thus, the transfer function (1) includes the transfer function of the motor  $G(p)$ :

$$G(p) = \frac{K}{1 + \tau p} \quad (3)$$

where  $K$  indicates the ratio between the thrust generated by the propeller and the signal pulse width supplied to the ESC. The time constant  $\tau$  indicates the response speed of the ESC and motor assembly, which is the time it takes to achieve the desired output signal from the arrival of the control signal. In fact, the parameters  $K$  and  $\tau$  are changed due to ignoring electrical factors, interference, changes in the parameters of components, or unwanted changes in motor torque.

Discretize the control object (1) and standard model (2) with a sampling period of  $T = 0.2$  s. We have the transfer function of the control object (4) and the transfer function of the standard model (5) in the discrete domain as follows:

$$H(z) = \frac{0,3031z + 0,08858}{z^2 + 0,4168z + 0.05848} \quad (4)$$

$$G_m(z) = (1 - z^{-1}) \left[ \frac{G_m(p)}{p} \right] = \frac{0.0615z + 0.0471}{z^2 - 1.341z + 0.449} = \frac{B_m}{A_m} \quad (5)$$

In fact, as analyzed above, the parameters in the transfer function of the control object are the transformation coefficients  $a_1, b_1, a_2, b_2$ , as in expression (6).

$$H(z) = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{B}{A} \quad (6)$$

At this point, the problem is reformulated as follows: Let the control object be the pitch channel of the quadrotor with a transfer function in the discrete domain of  $H(z)$ . Synthesis of a self-tuning adaptive controller combining STR-MRAC for quadrotor in the discrete domain so that the response of the system follows the standard model (5).

### C. Synthesis of an adaptive controller

*Step 1:* Build online estimation formulas for discrete transfer function parameters.

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} \quad (7)$$

$$\Rightarrow (1 + a_1 z^{-1} + a_2 z^{-2})Y(z) = (b_1 z^{-1} + b_2 z^{-2})U(z)$$

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 u(k-1) + b_2 u(k-2)$$

Set:

$$\begin{aligned} \varphi(k) &= [-y(k-1) \quad -y(k-2) \quad -u(k-1) \quad -u(k-2)]^T \\ \theta &= [a_1 \quad a_2 \quad b_1 \quad b_2]^T \\ \Rightarrow y(k) &= \varphi^T(k)\theta \end{aligned} \quad (8)$$

Algorithm to estimate parameters of a control object:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + L(k)\varepsilon(k)$$

$$\varepsilon(k) = y(k) - \varphi^T(k)\hat{\theta}(k-1)$$

$$L(k) = \frac{P(k-1)\varphi(k)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}$$

$$P(k) = \frac{1}{\lambda} \left[ P(k-1) - \frac{P(k-1)\varphi(k)\varphi^T(k)P(k-1)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)} \right]$$

In which:  $\hat{\theta}_0 = \text{rand}(4,1)$ ,  $p(0) = I_{4 \times 4}$

Running the online parameter estimation algorithm, at time  $k$ , we get:

$$\hat{\theta} = [\hat{a}_1 \quad \hat{a}_2 \quad \hat{b}_1 \quad \hat{b}_2] \quad (9)$$

*Step 2:* Design an indirect self-tuning controller according to the standard model, based on the parameters of the identification transfer function in step 1.

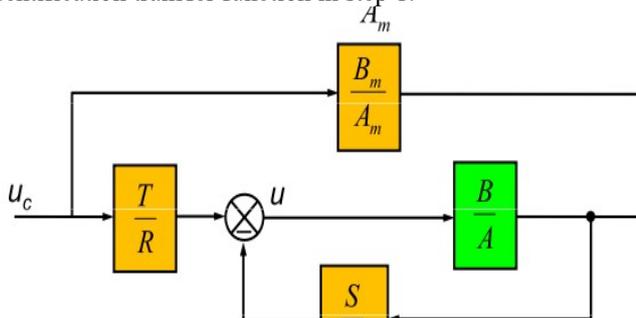


Fig.4. Control system structure diagram

Analyze B as:  $B = B^+ B^-$

Where  $B^+$  is a monic consisting of zeros located to the left of the complex plane.

$$\Rightarrow \begin{cases} B^+ = z + b_2 / b_1 \\ B^- = b_1 \end{cases} \quad (10)$$

Check the conditions for the existence of the controller according to the standard model:

$$B_m = B^- B_m' \Rightarrow B_m' = (0.0615z + 0.0471) / b_1 \quad (11)$$

$$\text{Order}(A_m) - \text{Order}(B_m) \geq \text{Order}(A) - \text{Order}(B)$$

To find  $A_0$ , proceed as follows:

$$\text{Order}(A_0) \geq 2. \text{Order}(A) - \text{Order}(A_m) - \text{Order}(B^+) - 1 = 0$$

$$\text{Choose Order}(A_0) \text{ to be } 0 \Rightarrow A_0 = 1$$

To find  $R_1$  and  $S$ , proceed as follows:

Choose Order ( $A_0$ ) as follows:

$$\text{Order}(R_1) = \text{Order}(A_0) + \text{Order}(A_m) - \text{Order}(A) = 0 + 2 - 2 = 0$$

$$\text{Order}(S) = \min \{ [\text{Order}(R_1) + \text{Order}(B^+) ], [\text{Order}(A_0) + \text{Order}(A_m) - \text{Order}(B^-)] \} = \min \{ [0 + 1], [0 + 2 - 0] \} = 1$$

$$\Rightarrow \begin{cases} R_1 = r_0 \\ S = s_0 q + s_1 \end{cases} \quad (12)$$

Calculate  $S$  and  $R_1$  by solving the diophantine equation:

$$AR_1 + B^- S = A_0 A_m$$

$$(z^2 + \hat{a}_1 z + \hat{a}_2)r_0 + \hat{b}_1(s_0 z + s_1) = z^2 - 1.341z + 0.449$$

$$\Rightarrow r_0 z^2 + (r_0 \hat{a}_1 + s_0 \hat{b}_1)z + (r_0 \hat{a}_2 + s_1 \hat{b}_1) =$$

$$= z^2 - 1.341z + 0.449$$

$$\Rightarrow \begin{cases} r_0 = 1 \\ s_0 = (-1.341 - \hat{a}_1) / \hat{b}_1 \\ s_1 = (0.449 - \hat{a}_2) / \hat{b}_1 \end{cases} \quad (13)$$

To find  $R$  and  $S$ , proceed as follows:

$$R = R_1 B^+ \Rightarrow \hat{R} = (z + \hat{b}_2 / \hat{b}_1) \quad (14)$$

$$T = A_0 B_m' \Rightarrow \hat{T} = (0.0615z + 0.0471) / \hat{b}_1 \quad (15)$$

## IV. SIMULATE AN ADAPTIVE CONTROLLER

The STR-MRAC combined self-tuning adaptive control system is designed in Figure 5.

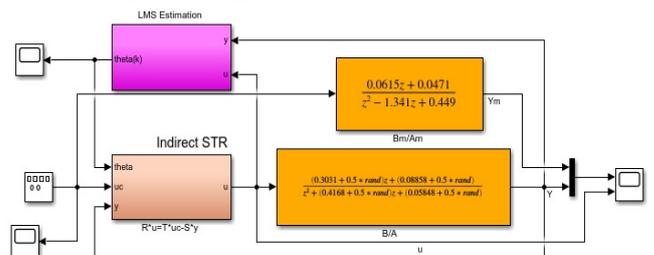


Fig.5. Diagram of a self-tuning adaptive controller for a quadrotor

Block B/A is the control object with parameters that vary according to the random rules of the rand function. The Bm/Am block is the standard model. The LMS Estimation block is a block that identifies the parameters of a control object. The indirect STR block is an indirect self-adaptive control block to control the system's output signal to always follow the standard model even when the parameters of the control object are changed.

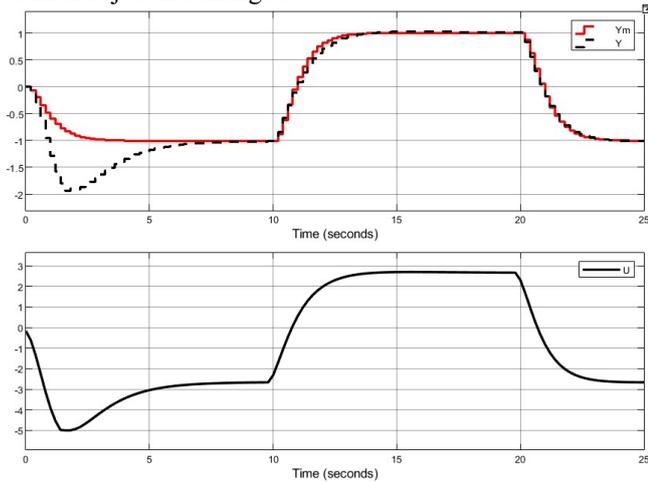


Fig.6. Simulation results of the self-tuning adaptive control system

In Figure 6, U is the form of the control signal at the input of the system. Ym is the form of the standard signal, and Y is the form of the system's output signal. Figure 6 shows that the system's output signal has tracked the standard signal.

The output response error of the adaptive controller is shown in Figure 7.

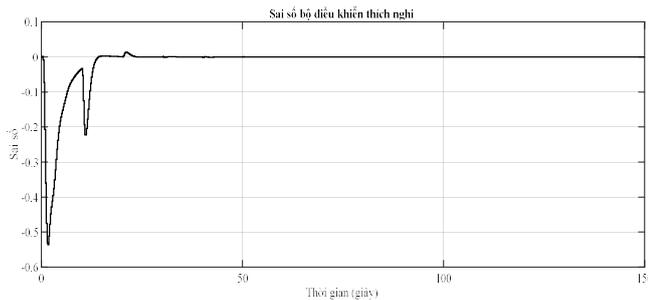


Fig.7. Error between the output response and the standard model when using an adaptive controller

Figure 7 shows the output response of the control object. It has followed the standard signal, with the output error gradually decreasing to 0. It can be said that the adaptive controller, after design, has achieved the criteria of quality requirements.

To more clearly demonstrate the superiority of the adaptive controller over the classical controller, examine the PI controller as shown in Figure 8.

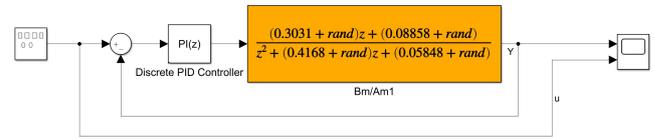


Fig.8. PI controller diagram for quadrotor

The simulation results in Figure 9 show that the output signal does not closely follow the input signal when using a PI controller for the same control object with variable parameters.

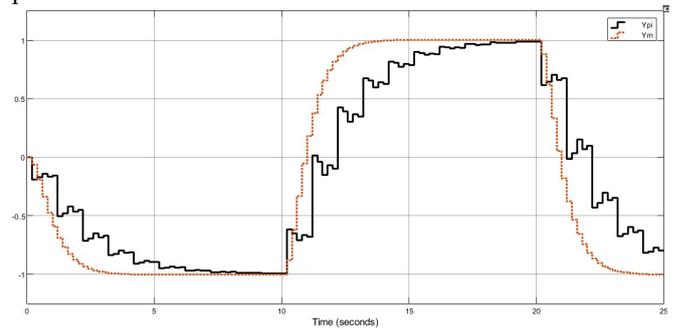


Fig.9. Simulation results of the PI controller for the quadrotor

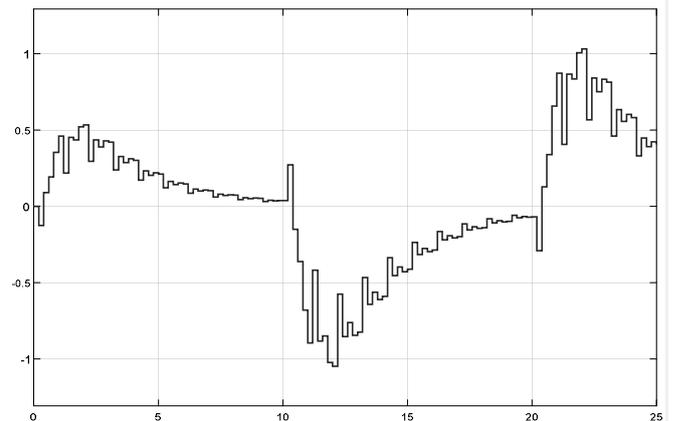


Fig.10. Error between the output response and the standard model when using a PI controller

Figure 10 shows that when a PI controller is used, the output response of the control object cannot closely track the standard signal, and there is always a non-zero output error. This shows that the adaptive controller excels at maintaining system stability, particularly under conditions of parameter variation.

## V. CONCLUSION

The research findings presented in this article establish that the utilization of a STR-MRAC adaptive controller in quadrotor systems is the optimal choice, offering several distinct advantages when compared to traditional PI-type controllers. Notably, when the control object's parameters vary randomly within permissible limits, the error between the control system's output response and the standard model remains minimal. This underscores the adaptive controller's exceptional ability to uphold system stability, especially in the face of parameter fluctuations.

The adaptive controller for quadrotors not only enhances control quality but also minimizes the impact of parameter variations on system stability. This holds significant implications for practical applications, particularly in dynamic and ever-changing operating conditions. The adaptive controller can swiftly adapt to these variations, thus ensuring the stability and control quality of quadrotor systems.

In summary, the outcomes of this study provide a robust foundation for implementing the STR-MRAC adaptive controller in quadrotor applications and similar systems. This contributes to the optimization of control quality and the assurance of system stability in fluctuating and challenging environments.

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