Research on the application of higher algebra knowledge in middle school mathematics

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Abstract—**Determinants, the concept of linear equations and generalized matrix theory and methods are an important part of the development of higher algebra. The rapid birth, development and maturity of these theoretical ideas and methods are always inseparable from the strong promotion and persistent practice of the early generation of mathematical researchers. On the contrary, using such new ideas and methods of higher algebra can easily, flexibly and quickly solve the common related problems in today's elementary mathematics practice. This paper mainly explores the application of determinant, matrix, Cauchy Schwartz inequality, quadratic form, the theory of solutions of linear equations in Higher Algebra in middle school mathematics.**

Index Terms—**Advanced algebra, determinant, system of linear equations, Cauchy Schwartz inequality, quadratic form.**

I. INTRODUCTION

Higher algebra education is not only the organic extension of mathematics knowledge in ordinary middle schools, but also the theoretical basis for learning modern engineering mathematics. Elementary mathematics and higher mathematics textbooks are closely related in educational ideas, methods and applications. Although higher mathematics is still difficult to be directly applied to the teaching practice of middle school mathematics, we can indirectly apply it to the teaching of middle school mathematics through the knowledge or thinking methods of higher algebra. Applying the knowledge of higher algebra to the teaching of middle school mathematics can make the middle school teachers' mathematics classroom full of high-tech and mystery, so as to improve the ability of mathematics teaching. Therefore, we should grasp the knowledge of Higher Algebra learned in University and make good use of them to make our middle school mathematics teaching more scientific, more comprehensive, more logical and more qualitative.

Although higher algebra knowledge is extremely complex compared with middle school mathematics, its content is extensive and its forms are diverse, and there are many places that can be applied to middle school mathematics. This paper preliminarily explores the application of determinant, matrix, Cauchy Schwartz inequality, quadratic form, the theory of solutions of linear equations and other knowledge in Higher Algebra in middle school mathematics, and hopes to get the resonance of readers and trigger more thinking.

II. THE APPLICATION OF DETERMINANT IN MIDDLE SCHOOL MATHEMATICS

There are many applications of determinants. Here we only briefly introduce the application of determinants in the factorization of polynomials. The concept of factorization of polynomials has been discussed in detail in higher algebra, and will not be repeated here. Determinants are factorizations that can be directly applied to polynomials. From the knowledge learned, generally speaking, the value of the determinant can be calculated, and an equation can be used to express the determinant. The process of factorization is actually the process of decomposing a known formula into another equal equation, which shows that we can first construct such a determinant corresponding to it, and then disassemble it differently through the properties of the determinant to solve the problem [1-5].

Example 1. Factorize the polynomial
$$x^3 + y^3 + z^3 - 3xyz .$$

Solution. ① Method 1. Based on the factorization method of middle school mathematics, the specific process is as follows:

$$x^3 + y^3 + z^3 - 3xyz$$
$$= (x+y)^3 + z^3 - 3xyz - 3xy^2 - 3x^2 y$$
$$= (x+y+z)[(x+y)^2 - (x+y)z + z^2] - 3xy(x+y+z)$$
$$= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx).$$

② Method 2. Based on the solution of determinant, the specific process is as follows.

Let
$$D = \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix},$$

then we have $D = x^3 + y^3 + z^3 - 3xyz$. From the properties of determinant, we obtain

$$D = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ y & z & x \end{vmatrix}.$$

Thus we have

$$D = (x+y+z)\begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ y & z & x \end{vmatrix}.$$

Expand it according to the first line, and we get

$$D = (x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

Through determinant, the polynomial can be clearly transformed, so it is easier to solve the problem than directly transforming the polynomial. However, when using determinants to solve problems, we should pay attention to mastering determinants well is the premise. However, determinants are novel for middle school students. Properly teaching the knowledge of determinants can greatly improve students' enthusiasm in mathematics learning, so as to correctly guide students to think from multiple perspectives and guide them to look at the knowledge they have learned from different perspectives.

III. THE APPLICATION OF MATRIX IN MIDDLE SCHOOL MATHEMATICS

Matrix is widely used in middle school mathematics, such as solving first-order equations, finding plane normal vectors, and so on. Here, we use an example to study the application of matrix in finding the general term formula in middle school.

Example 2. Given $m_0 = 1$, $m_1 = 1$, $m_{i+1} = m_i + m_{i-1}$ in the sequence $\{m_i\}$, find the general term formula of the sequence $\{m_i\}$.

Solution. First, $m_{i+1} = m_i + m_{i+1}$ is represented by a matrix:

$$\begin{bmatrix} m_{i+1} \\ m_i \end{bmatrix} = \begin{bmatrix} m_i + m_{i-1} \\ m_i \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} m_i \\ m_{i-1} \end{bmatrix}.$$

Let

$$M_i = \begin{bmatrix} m_{i+1} \\ m_i \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix},$$

then $M_i = AM_{i-1}$, and

$$M_0 = \begin{bmatrix} m_1 \\ m_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Thus we have

$$M_1 = AM_0, M_2 = AM_1 = A^2 M_0, \Lambda, M_n = A^n M_0.$$

Since

$$A = P\begin{bmatrix} \dfrac{1+\sqrt{5}}{2} & \\ & \dfrac{1-\sqrt{5}}{2} \end{bmatrix}P^{-1},$$

where

$$P = \begin{bmatrix} \dfrac{1+\sqrt{5}}{2} & \dfrac{1-\sqrt{5}}{2} \\ 1 & 1 \end{bmatrix},$$

$$P^{-1} = \begin{bmatrix} \dfrac{1}{\sqrt{5}} & -\dfrac{1-\sqrt{5}}{2\sqrt{5}} \\ -\dfrac{1}{\sqrt{5}} & \dfrac{1+\sqrt{5}}{2\sqrt{5}} \end{bmatrix},$$

so we get

$$A^n = P\begin{bmatrix} \dfrac{1+\sqrt{5}}{2} & \\ & \dfrac{1-\sqrt{5}}{2} \end{bmatrix}^n P^{-1},$$

and

$$M_n = \begin{bmatrix} m_{n+1} \\ m_n \end{bmatrix} = A^n M_0$$

$$= \frac{1}{\sqrt{5}}\begin{bmatrix} \left(\dfrac{1+\sqrt{5}}{2}\right)^{n+2} & -\left(\dfrac{1-\sqrt{5}}{2}\right)^{n+2} \\ \left(\dfrac{1+\sqrt{5}}{2}\right)^{n+1} & -\left(\dfrac{1-\sqrt{5}}{2}\right)^{n+1} \end{bmatrix}.$$

Therefore, we have

$$m_n = \frac{1}{\sqrt{5}}\left[\left(\dfrac{1+\sqrt{5}}{2}\right)^{n+1}, -\left(\dfrac{1-\sqrt{5}}{2}\right)^{n+1}\right].$$

In the above example, the problem is transformed more intuitively with the matrix, and the elementary properties of the matrix are applied, so it is easy to solve. However, it should be noted that the ability to calculate data is a test, so only those skilled in matrix can use this kind of method to solve. Similarly, this method is challenging and attractive for middle school students, and can appropriately guide students to think and learn.

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IV. APPLICATION OF CAUCHY SCHWARTZ INEQUALITY IN MIDDLE SCHOOL MATHEMATICS

Cauchy-Schwartz inequality is one of the most important inequalities in higher algebra teaching, and it is also widely used in junior high school mathematics.

Theorem In Euclidean space V, for all $\xi, \eta \in R^n$, it holds that

$$(\xi, \eta) \leq |\xi| \cdot |\eta|,$$

and the equal sign holds if and only if ξ and η are linearly correlated.

Under the standard internal intersection product, the expression of Cauchy Schwartz inequality is as follows

$$(a_1 b_1 + a_2 b_2 + ... + a_n b_n)^2$$
$$\leq (a_1^2 + a_2^2 + ... + a_n^2)(b_1^2 + b_2^2 + ... + b_n^2),$$

and the equal sign holds if and only if (a_1, a_2, Λ, a_n) and (b_1, b_2, Λ, b_n) are linearly correlated, i.e., $a_1 : b_1 = a_2 : b_2 = \Lambda = a_n : b_n$.

Example 3. Let x, y, z be positive numbers, and $x + y + z = 1$, prove that $\dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c} \geq 9$.

Proof. In R^3, we use standard inner product. Let

$$\xi = (\sqrt{a}, \sqrt{b}, \sqrt{c}), \eta = \left[\frac{1}{\sqrt{a}}, \frac{1}{\sqrt{b}}, \frac{1}{\sqrt{c}}\right],$$

then we have

$$|\xi|^2 |\eta|^2 = (a + b + c)(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}) = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

Therefore, we get

$$(\xi, \eta)^2 = \left(a \cdot \frac{1}{a} + b \cdot \frac{1}{b} + c \cdot \frac{1}{c}\right)^2 = 9.$$

From Cauchy inequality, we have

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq 9,$$

and the equal sign holds if and only if ξ and η are linearly related.

Based on the above problems, we know that the prerequisite for the proper use of Cauchy-Schwartz inequality is to construct a suitable Euclidean space, especially to construct the inner product calculation. In addition, you must pay attention to finding two suitable spatial vectors.

Although Cauchy-Schwarz inequality is more complex in the field of higher algebra, it is simple and easy to operate when applied to elementary mathematics, and Cauchy-Schwarz inequality can also be applied to the problem of solving the maximum value of simple formulas. However, the answer from this example also reveals that what we need in mathematics teaching is a heart that is good at observing and daring to think. Therefore, here we can clearly see that a variety of solutions to a problem is not our ultimate goal. It is crucial to achieve the organic conclusion and application of various knowledge, so as to further reflect the necessary comprehensive quality of a real middle school mathematics teacher.

V. THE APPLICATION OF QUADRATIC FORM IN MIDDLE SCHOOL MATHEMATICS

Quadratic form is very important in higher algebra, but in middle school mathematics, quadratic form knowledge can also be used to solve middle school mathematics problems. Next, let's look at the application of quadratic form in finding the maximum and minimum value in middle school mathematics.

Theorem Let $f(x) = x'Ax$ be a quadratic form, then the maximum (minimum) value of $f(x)$ under the condition $\sum_{i=1}^{n} x_i^2 = 1$ is exactly the maximum (minimum) eigenvalue of matrix A.

Example 4. Let $f(x) = x^2 + 2xy + 3y^2$, and it holds that $x^2 + y^2 = 1$. Find the maximum and minimum value of $f(x)$.

Solution. The matrix of quadratic form $f(x)$ is

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}.$$

From

$$|\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 \\ -1 & \lambda - 3 \end{vmatrix} = \lambda^2 - 4\lambda + 2,$$

we have $\lambda_1 = 2 + \sqrt{2}$, $\lambda_2 = 2 - \sqrt{2}$.

Therefore, according to the above theorem, the maximum value of $f(x)$ under $x^2 + y^2 = 1$ is $2 + \sqrt{2}$ and the minimum value is $2 - \sqrt{2}$. [5] examined the development and refinement of possible mathematical models for the intellectual system of career guidance. Mathematical modeling of knowledge expression in the career guidance system, Combined method of eliminating uncertainties, Chris-Naylor method in the expert information system of career guidance, Shortliff and Buchanan model in the expert information system of career guidance and DempsterSchafer in the expert information system of career guidance method has been studied. The algorithms of the above methods have been

developed.

Through the transformation of polynomials, we get quadratic form, and then get the matrix of quadratic form. Through the application of the theorem, we will get the answer to the question. It's actually very simple to solve this problem by using quadratic form, but for middle school students, they haven't touched it yet. Students can be properly guided and explained to cultivate their thirst for knowledge.

In addition, the theory of quadratic form is very important, which can be applied to conic and quadric equation. Quadratic theory can help students understand problems properly and adapt to the complex content of high school mathematics.

VI.   APPLICATION OF LINEAR EQUATIONS THEORY IN MIDDLE SCHOOL MATHEMATICS

When using the theory of linear equations to solve problems, we need to use the theory of linear equations to analyze and deal with the problems, and construct a model of linear equations. The better the construction research is done, the easier the problem will be solved.

Example 5. Given the function $f(x) = x^2 + ax + b$, prove that at least one of $|f(1)|, |f(2)|, |f(3)|$ is not less than $\dfrac{1}{2}$.

Solution. Substitute $x = 1, 2, 3$ into the function expression and sort it out as follows:

$$\begin{cases} a + b + (1 - f(1)) = 0, \\ 2a + b + (4 - f(2)) = 0, \\ 3a + b + (9 - f(3)) = 0. \end{cases}$$

This is a system of homogeneous linear equations about a, b, 1, and it has non-zero solutions, therefore,

$$\begin{vmatrix} 1 & 1 & 1 - f(1) \\ 2 & 1 & 4 - f(2) \\ 3 & 1 & 9 - f(3) \end{vmatrix} = 0,$$

thus we have $f(1) - 2f(2) + f(3) = 2$. Assume that the conclusion is not true, that is

$$\left| f(1) \right| < \frac{1}{2}, \quad \left| f(2) \right| < \frac{1}{2}, \quad \left| f(3) \right| < \frac{1}{2},$$

Then $2 < f(1) - f(2) + f(3) < 2$. This is obviously contradictory, so the proposition is true.

Example 5 first transform the topic, and then according to the meaning of the topic, use the properties of linear equations, that is, when the homogeneous linear equations have non-zero solutions, the value of the corresponding coefficient determinant is 0. Then through the method of counter evidence, draw a conclusion. When solving problems, we must pay attention to flexible thinking and clear thinking.

Linear equations are widely used in middle school mathematics, especially in solving multiple equations. Proper application of the theory of linear equations can improve the speed of solving equations. Therefore, the theory of linear equations should be better applied to middle school mathematics, but it should be noted that mathematics teachers should simply tell it so that students can understand it easily. Students must not be forced to fully master it, as long as it can assist in problem solving.

In short, linear equations are widely used, and we still need to work together to apply them to middle school mathematics.

REFERENCES

[1] R.Y. Yang, W.Jiang. Application of higher algebra theory in polynomial decomposition. Journal of Tangshan Normal University, 2006,28 (05): 33-34.
[2] J.B. Li, D.P. Yu Learning higher mathematics knowledge is an important way to improve the ontological knowledge of middle school mathematics teachers. Chongqing: Journal of School of mathematics and finance, Chongqing University of Arts and Sciences, 2017:28-30.
[3] F.T. Cao. Some applications of Higher Algebra in middle school mathematics. Journal of Guangxi Normal University (PHILOSOPHY AND SOCIAL SCIENCES EDITION), 2006,27:135-137.
[4] R.K. Zhu. Research on middle school mathematics problems from the perspective of higher algebra. Journal of Jimei University, 2009,10 (4): 77-80.
[5] Christo Ananth, A.R. Akhatov, D.R. Mardonov, F.M. Nazarov, T. AnanthKumar, "Possible Models and Algorithms for the Intellectual System of Professional Direction", International Journal of Early Childhood Special Education, Volume 14, Issue 05, 2022,pp. 4133-4145.

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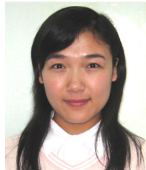
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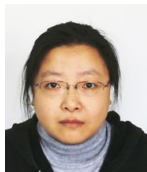
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