



Some Discussions on Teaching Numerical Analysis

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Abstract—Numerical analysis is a specialized course in the mathematics department of the university. In this paper, we discuss the characteristics of numerical analysis and the three aspects of teaching numerical analysis - classroom teaching, computer experiments, and examinations. We believe that in the teaching process, we should not only make full use of classroom teaching to introduce the practical value of numerical analysis knowledge, but also go further to cultivate students' mathematical thinking and scientific research ability through the reform of on-line experiments and assessment methods.

Index Terms—University education, Numerical analysis, Classroom teaching, Modern teaching methods.

I. INTRODUCTION

With the development and popularization of computer technology, after the experimental and theoretical methods of scientific research, scientific computing has become the third important means of scientific research, which has been widely used in the fields of physics, mechanics, chemistry, life science, astronomy, economic science and sociology. At the same time, because the scientific computing method is based on the existing experimental data, through the analysis, mining its intrinsic regularity to achieve the understanding of the object of study, and the cost of scientific computing to study and discover the intrinsic laws is much lower than the cost of experiments, so scientific computing has become an indispensable tool in modern science and technology research. Therefore, numerical computation methods and data processing methods applicable to computer processing, as an important part of scientific computing, have gradually become a mandatory course for undergraduate and graduate students in science and technology[1-3].

The most obvious difference between this course and other mathematics courses is the perfect combination of theory and practice, which contains both rigorous mathematical theory and has a strong practical value. Numerical analysis is taught as a university course in mathematics, but we need to emphasize both its theoretical structure and its practical value. Students' ability to apply mathematics can be greatly improved by teaching a good numerical analysis course.

II. COURSE FEATURES OF NUMERICAL ANALYSIS

The central task of teaching numerical analysis is to introduce the methods of data processing suitable for computer processing by using the obtained discrete

experimental data in the solution of mathematical models corresponding to deterministic phenomena, and the corresponding solution process is transformed into algorithms for data processing containing only four regular operations and logical operations. Therefore, the core of numerical analysis teaching is the algorithm (or method) of various data processing and the corresponding algorithm analysis. The algorithm analysis includes the feasibility (i.e. convergence or stability) analysis, validity analysis and error analysis of data processing methods. And providing effective and stable data processing methods is the basic purpose of numerical analysis calculation.

A. Focus on methodological and practical aspects

Many of the methods in numerical analysis are based on the theoretical foundations learned in advanced mathematics, but unlike the emphasis on theoretical analysis in advanced mathematics, it focuses more on how to apply the results of these theoretical analyses. For example, the difference quotient numerical differentiation formula is the application of Taylor series in higher mathematics, the function interpolation integrates the concept of function space in linear algebra and the differential median theorem in higher mathematics, and the catch-up method uses the matrix decomposition theory in linear algebra. The Numerical Analysis course is a more application-oriented science, with special attention to the balance between methodological precision and effectiveness. For example, when doing function interpolation, there are various methods available: Lagrange type, Newton type, Hermite type, spline type interpolation, etc. In specific use, Lagrange type interpolation is simple, rapid, suitable for point interpolation fast calculation; Newton type interpolation has inheritance, suitable for extrapolation calculation; Hermite type interpolation has relatively Newton type interpolation is inheritable and suitable for extrapolation; Hermite type interpolation has high continuous differentiability, but is computationally intensive; spline type interpolation also has high smoothness, but requires additional constraints. If students have mastered the various methods and understand their characteristics, they will be able to work with half the effort.

B. Focus on the use of computers

Although the methods presented in numerical analysis are general, many problems can only be implemented on a computer. For example, solving a system of linear algebraic



equations with 100 equations is not possible with the brain and hands alone, no matter how good the computational methods are. This requires that we be able to program the calculations using a computer or use off-the-shelf software to perform the calculations. Whether programming or using software, students are required to be computer literate, and a qualified student should learn to use computers to solve problems they encounter. In addition, many computational methods have computer-friendly features, such as loop solving and iteration, which are complex methods for humans but only require very simple programming for computers [4-5].

III. SEVERAL ISSUES THAT SHOULD BE NOTED IN ACTUAL TEACHING

A. Classroom lectures

Classroom lectures are the most important part of numerical analysis teaching, and we should use this part as much as possible to highlight the characteristics of numerical analysis courses. As the problems involved in numerical analysis are extracted from the real world, then the mathematical theory is applied to derive and finally propose specific solutions. Therefore, the teaching of each numerical method should try to present problems from examples, guide students to think about how to use mathematical knowledge to construct solutions, and then give the corresponding mathematical theory. In fact, the teaching process of numerical analysis is a simple scientific research process. This method of teaching can stimulate students' interest in learning and make their knowledge more solid.

At the same time, in the teaching, we should also pay attention to the structure of the course of numerical analysis. When students are first introduced to this course, they often find it difficult, even more difficult than mathematical analysis, and they find it difficult to follow the teacher's lecture ideas. Over time, students' interest in learning gradually decreases. The main reason for this situation is that the knowledge of numerical analysis is composed of many different parts, and the structure is relatively loose. Without a good thinking backbone throughout the course, students can only accumulate mixed knowledge after learning the whole course, but cannot understand the connection between knowledge structures in a deeper level. Therefore, teachers are required to make an effort to maintain a close connection between the chapters in the course of teaching the course, and to explain that the general idea of numerical analysis is "approximate solution".

Mathematics is very logical, and numerical analysis is no exception. In the process of teaching, the teacher should ask questions in a progressively deeper way, leading students to see the connections and differences between the various points of knowledge. When a new problem is posed, students are inspired to think about why the problem was posed and where the flaws in the original method lie. In this way, the

students' thinking will be clearer. For example, in the teaching of interpolation, the problem of uniqueness of the existence of the solution of the interpolating polynomial is solved first, and then the expression is solved, which leads to the Lagrange interpolation method. By analyzing its properties, it is found that the computational effort increases after adding nodes, which further leads to Newton interpolation to save computational effort. Then, we introduce Hermite interpolation and segmental interpolation by Runge phenomenon, and finally, we give the sample interpolation, so that the knowledge of interpolation forms a line.

B. Adding "basic formulation" to the teaching process

The teaching content of numerical analysis course is based on the basic knowledge of advanced mathematics and linear algebra, and provides suitable data processing methods for the purpose of understanding objective laws in the face of the data presented by realistic deterministic phenomena. On the surface, the teaching content is loose and does not have a compact structure, which makes it difficult for teachers to organize teaching and students to master. However, if teachers grasp the core of the data processing process, which is to provide a stable (or convergent) and effective algorithm, and increase the introduction of the basic formulation of various algorithms in practical use, the problem of "difficult to organize teaching and learning, difficult to master" can be solved.

For example, in the teaching of Gauss-type product formula, in addition to the general formulation of the product formula, the idea of "good point set" is added to improve the level of introduction. The so-called Gauss-type product formula means that in a system (such as an immune system or a defense system), if the number of nodes is known, how to choose the distribution and strength of the nodes to make the system's goal more effective. The result is that the student can grasp the essence of this abstract concept, so that he/she can find the relevant combination of the concept from different disciplines to facilitate the mastery of the idea.

C. Hands-on experiment

Numerical analysis is most clearly distinguished from other mathematics courses by its hands-on laboratory sessions. The purpose of this class is to develop students' practical and programming skills and to apply the theory of numerical analysis learned in class to concrete examples. It is a process of digesting the knowledge learned in class. Students can try different methods to solve the same problem and compare them to verify the advantages and disadvantages of each method. Problems in numerical analysis are difficult to understand by classroom teaching and theoretical derivation alone. For example, students can deepen their understanding of classroom knowledge by drawing diagrams or comparing tables in the process of actual computation, such as convergence and stability.

The combination of hands-on experiments and classroom



teaching is also a kind of primary research process, which gives students an initial experience of the fun, difficulties and atmosphere of research. It should be noted, however, that this part of the teaching process is also very problematic - the repetition of practical topics and the problem of plagiarism among students. It is possible to try to group students to do different problems, or to apply multiple methods to the same problem, choosing different parameters to solve it, and then compare the differences between them. The results of this approach are very satisfactory, but there is still much room for improvement in the teaching of the hands-on part of the experiment.

D. Assessment and supervision of learning methods

Numerical analysis is a mathematics course, so doing more exercises is one of the most direct means to learn mathematics. Many of the theoretical derivations in the textbook can not be understood just by thinking about them, and it is difficult to understand the contents of the book without doing a certain amount of exercises. We know that teaching is a two-sided process, of course, the students' own efforts are essential, but it is not realistic for most students to rely solely on their self-motivation. Especially in the modern business climate, learning mathematics is a choice most students have to make. Therefore, it is necessary to use some necessary tools, such as regular homework review and proper testing and recording of the results in the grades.

The examination is the last part of each course, and it is necessary to set it reasonably. For the characteristics of numerical analysis courses, in addition to the usual grades, numerical experiments should be included in the scope of assessment, and the written report on the computer is one of the assessment contents. In addition, we can try the combination of oral and written examinations. Students should tell the teacher the method, theory and results of numerical analysis used in a certain problem, and further analyze the differences between various methods. This approach is very effective, as it urges the students to examine in detail the knowledge of the textbook and the data results of the computer tests, which deepens the understanding of the textbook content.

IV. CONCLUSION

Numerical analysis course as part of the basic mathematics course, it is very different from other mathematical analysis courses, it emphasizes the emphasis on computer applications,

focusing on methods and engineering ideas to solve practical problems, it is these methods and ideas make it play a fundamental role in the mathematics laboratory courses and mathematical modeling teaching. Of course, the numerical analysis course is not a panacea, for example, the discrete problems in numerical analysis course are very different from the discrete problems in discrete mathematics, and a considerable part of the content of operations research is not covered in numerical analysis course. Since numerical analysis courses teach students many new ideas and methods, students do not feel a huge obstacle in learning other new things. If teachers emphasize the practical aspects of numerical analysis and the engineering aspects of numerical analysis in their courses, they will be able to educate students in the application of mathematics with half the effort.

REFERENCES

- [1] L.-J. Li, *Numerical calculation method*, 2st ed. Beijing: Beijing University of Technology Press, 2006.
- [2] Z. Guan and J.-P. Lu, *Fundamentals of Numerical Analysis*. Beijing: Higher Education Press, 1998.
- [3] T. Li, Y.-J. He, Z.-J. Liu, *MATLAB Toolbox Application Guide- -Applied Mathematics*, Beijing: Electronic Industry Press, 2000.
- [4] L.-R. Chen, *Introduction to Mathematical Modeling*, Beijing: Beijing University of Posts and Telecommunications Press, 2000.
- [5] L. Zhang and B. Zhang, "Good point set genetic algorithm", *Journal of Computer Science*, vol. 24, pp. 917-922, September 2001.

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