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Discussion on the Experimental Aspects of Numerical Analysis Teaching

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***Abstract*—Lack of intuitive and in-depth understanding of algorithm theory and the disconnection between theory and application are the problems faced by the current numerical analysis practical classes. Algorithms are the core of the practical teaching of numerical analysis. We have designed some comprehensive experiments with the fitting algorithm of recursive equations and the Lagrange interpolation algorithm as examples, and studied how to effectively carry out practical teaching in numerical analysis courses.**

***Index Terms*—Numerical analysis, the Lagrange interpolation algorithm, the fitting algorithm, the practical teaching.**

I. Introduction

Numerical analysis, also known as numerical computational methods, is concerned with the theory and corresponding methods of approximate solutions using computers with finite number of effective bits. It aims to achieve the solution of a given mathematical problem with the least possible consumption of computational resources, while taking into account the error, convergence and stability of the numerical algorithm [1-3]. The content of the numerical analysis course is mainly the extension and development of knowledge or models from higher mathematics and linear algebra in the study of practical problems. It includes the basic contents of numerical algebra, numerical calculus, numerical approximation and numerical solution of differential equations etc. Numerical analysis, as an important branch of mathematics, has rich contents and profound research methods. It has the characteristics of high abstraction and rigor of pure mathematics, as well as the characteristics of wide application and high technicality of experiment. It is a high practical mathematics course closely integrated with the use of computers.

It has become a consensus to carry out practical teaching in the teaching of numerical analysis courses. The traditional teaching mode focuses more on the proof of theorems and the derivation of computational formulas, and students often have difficulties in understanding, lack of intuition and in-depth understanding of algorithmic theory, and the theory is often disconnected from the application, and often students still do not know how and where to use the methods in numerical analysis after learning. Many schools have made useful attempts on how to effectively use the limited laboratory time to make students really master the fundamentals of numerical analysis and how to get more time to guide students to explore and experiment in depth [4-6]. At present, many universities have introduced MATLAB software in the teaching of numerical analysis and used MATLAB to write numerical algorithm programs, but it is still mainly lecture-based, with the teacher elaborating the basic theory and basic methods and verifying the basic methods through software, which does not have the expected effect on the improvement of students' ability [7]. In order to make students more interested in learning numerical analysis and have better learning effect, we have designed some comprehensive experiments combined with the content and characteristics of numerical analysis course teaching given below briefly.

II. Several Comprehensive Experiments

A. *Fitting of recursive equations*

Example 1. Let $\alpha_{n+1} = \alpha_n + 1/\alpha_n$, $\alpha_0 = 1$ .For some n, draw a scatter plot of (n, α_n), and find a suitable function to fit the above graph.

Solution: Since the equation $\alpha_{n+1} = \alpha_n + 1/\alpha_n$ is a kind of immobile point iteration, we first use the loop command in MATLAB software to calculate the first 5000 values and plot the $(\boldsymbol{n}, \boldsymbol{\alpha_n})$ scatter plot in figure 1. The MATLAB program is as follows

```
x=1;
n=999;
y=zeros(1,n);
y(1)=1
for t=1:n
x=x+1/x;
y(t+1)=x;
end
n=1:1000;
plot(n,y);
```

From the figure 1, it can be seen that α_n is an increasing function of n. We use the index model $y_n = an^b$ to perform a least squares fitting. For the $y_n = an^b$, we take the logarithm at both ends simultaneously yields, then we get

$$\ln y_n = \ln a + b \ln n.$$

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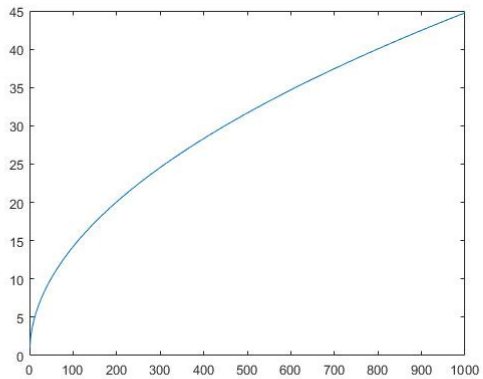

Fig. 1. Scatter plot of (n, α_n)

Let $\overline{y_n} = \ln y_n, A = \ln y_n, x = \ln n$, then the model becomes a primary polynomial $\overline{y} = A + bx$. Following this idea, the MATLAB program is designed as follows.

```
x=log(n)
yg=log(y);
p=polyfit(x,yg,1)
```

The output of the command after execution is b= 0.4993, A=0.3530, a=e^A=1.4234. The fit function is obtained as $y_n = 1.4234n^{0.4993}$.The fitting effect is shown in figure 2.

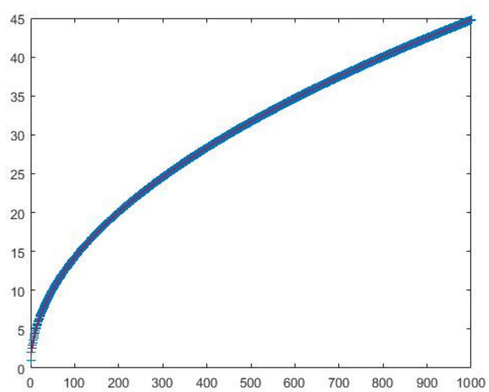

Fig. 2.the fitting effect

In the fitting function

$y_n = 1.4234n^{0.4993}$, b=0.4993 $\approx$ 0.5, $1.4234 \approx \sqrt{2}$.

We get $y_n \approx \sqrt{2n}$. Let $\alpha_n \approx \sqrt{2n}$ thus we obtain the approximate relationship equation

$$\sqrt{2n(n+1)} \approx \sqrt{2n} + \frac{1}{\sqrt{2n}}$$

The squared error can be obtained by squaring both ends of the equal sign of the above equation as 1/n. Obviously the larger the value of n the higher the degree of approximation of the formula.

B. Experimental design of the Lagrange interpolation method

Example 2: There is an iron and steel plant with an annual production of 330,000 tons, and now it is necessary to count its production costs, but it is no longer possible to do so because various data information of the plant is partially lost. In such a situation, the statistics department collects data from five steel plants that are very close to this plant in terms of personnel, equipment, production capacity, etc., as shown in Table 1. How can we estimate the production cost of the plant more accurately based on the information of the known 5 steel plants?

TABLE I: THE STEELPLANT STATISTICS

Steel plants	A	B	C	D	E
Production (million tons)	30	32	34	36	38
Production costs (billion yuan)	6.0	6.2	6.5	6.6	6.9

We can use The Lagrange interpolation method to construct an interpolating polynomial to solve such a problem. Let the steel plant have almost the same personnel, equipment, production capacity and other aspects as the five steel plants in Table 1. We use x_i and y_i to refer to the output and production cost of the steel plant, respectively, then we can obtain the interpolated node information, as shown in Table 2.

TABLE II: THE INTERPOLATION NODE INFORMATION

i	0	1	2	3	4
x_i	30	32	34	36	38
y_i	6.0	6.2	6.5	6.6	6.9

Using the 5 nodes, a quadratic the Lagrange interpolation polynomial can be constructed as follows

$$P_4(x) = \sum_{k=0}^{4} y_k l_k(x)$$

$$\boldsymbol{l_k(x) = \frac{(x-x_0)\cdots(x-x_{k-1})(x-x_{k+1})\cdots(x-x_4)}{(x_k-x_0)\cdots(x_k-x_{k-1})(x_k-x_{k+1})\cdots(x_k-x_4)} (k=0,1,2,3,4)}$$

When writing the MATLAB program, first write the M file for the Lagrange algorithm with the following code

```
function [C,L1,l] = lagran1(X,Y)
m = length(X); L = ones(m,m);
for k = 1 : m
   V = 1;
   for i = 1 : m
      if k ~= i
         V = conv(V,poly(X(i))) / (X(k) - X(i));
      end
   end
   L1(k, :) = V; l(k, :) = poly2sym(V);
end
C = Y * L1;
L = Y * l;
```

Second, for Example 2, let X=[30:2:38];Y=[6.0 6.2 6.5 6.6.6.9]; running the M file yields the polynomial.

$P_4(x) = 0.0018x^4 - 0.2469x^3 + 12.5052x^2 - 280.6875x + 2361$

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The final calculation yields $1 billion. Figure 3 shows the image of 4 times Lagrange interpolation.

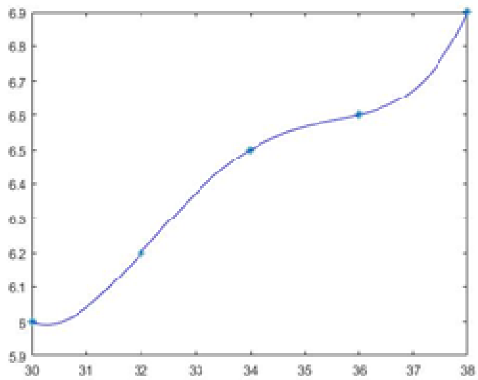

Fig. 3.the Image of the Lagrange interpolation polynomial

The case study enables students to deepen their understanding of the principle of Lagrange interpolation and its application, and improves their hands-on ability to solve practical problems.

III. Conclusion

Numerical analysis is the study of numerical computation methods for various mathematical models, and it is oriented to computers. This determines that numerical analysis classes cannot focus only on theory, like other mathematics classes. Without numerical experiments students cannot deeply understand the nature of numerical analysis. The two numerical analysis experiments designed in this paper are the deepening and application of relevant basic knowledge, which help students deepen their understanding of the numerical methods they have learned. By completing the experiments, students can effectively improve their scientific computing skills.

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