ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE)
Vol. 8, Issue 4, April 2022

Common Interpolation Methods and Their Applications

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***Abstract*—Interpolation is an important part of the field of numerical analysis. In this paper, we discuss the central and important role of interpolation in numerical analysis. We compare the advantages and disadvantages between interpolation methods and give a brief description of the application of interpolation methods in modern technology.**

***Index Terms*—Numerical analysis, Interpolation methods.**

I. INTRODUCTION

In many practical problems, some functions $f(x)$ have analytical expressions, but their calculation is complicated and inconvenient to use. Even some functions $f(x)$ only give the value of the function at certain points. Therefore, we want to construct a simple function $P(x)$ to approximate $f(x)$ based on the given information, which can reflect the properties of the function and is easy to calculate. We can use the interpolation method to achieve this purpose [1-3].The interpolation method is the basis of numerical analysis such as numerical calculus and numerical solution of differential equations. Today, with the widespread use of computers, the interpolation method is combined with programs such as MATLAB to play a wider role, so that interpolation and fitting can be solved up to hundreds or even thousands of times (if the computer allows). For this reason, a large number of mathematicians and software editors have devoted themselves to numerical analysis and mathematical software, leading to an unprecedented development of the interpolation method.

II. COMMON ALGEBRAIC INTERPOLATION FORMULAS AND THEIR CONSTRUCTION

A. *Definition of interpolation*

The interpolation method is a method to find the approximate expression $P(x)$of the function $f(x)$ based on a set of data, as shown in Table 1.

TABLE I: INTERPOLATION DATA SHEET

x_i	x_0	x_1	x_2	…	x_n
$f(x_i)$	$f(x_0)$	$f(x_1)$	$f(x_2)$	…	$f(x_n)$

The necessary condition for the interpolation method is that the error function or the residual term $R(x) = f(x) - P(x)$ satisfies the relation

$$R(x_i)=f(x_i)-P(x_i)=0(i=0,1\ldots,n).$$

When the interpolation function $P(x)$ is polynomial, it is called algebraic interpolation method. The algebraic interpolation methods include Lagrange interpolation, successive linear interpolation, Newton interpolation, Hermite interpolation, segmental interpolation, and spline interpolation. The basic idea is to use higher algebraic polynomials or segmented lower polynomials as approximate expressions of the interpolated function $P(x)$.

B. *Lagrange interpolation*

The nth Lagrange interpolation polynomial $P_n(x)$ for the data points in Table 1 can be expressed as

$$P_n(x)=\sum_{i=0}^{n} y_i l_i(x)=\sum_{i=0}^{n} y_i\left(\prod_{\substack{j=0\\ j\neq i}}^{n}\frac{x-x_j}{x_i-x_j}\right)$$

where $l_i(x)$ $(i=0,1\ldots,n)$ is called the interpolation basis function. The interpolation residual term is

$$R(x)=f(x)-P_n(x)=\frac{f^{(n+1)}(\zeta)}{(n+1)!}(x-x_0)(x-x_1)\cdots(x-x_n)$$

here $\zeta\in(a,b)$.

Lagrange interpolation polynomials are convenient for theoretical derivation and formal description of the algorithm, but are not convenient for calculating the function values. When using the Lagrange interpolation polynomial $P_n(x)$ to calculate the approximation of the function, if the accuracy is not satisfied and additional nodes are needed, then the original calculated data cannot be reused. To overcome this drawback, the successive linear interpolation method or the Newton interpolation polynomial method is usually used. These two methods are based on Lagrange interpolation to combine known computational values, which can improve the computational efficiency and achieve the effect of accelerated computation.

C. *Successive linear interpolation method*

The formula for successive linear interpolation is the following.

$$I_{0,1,\cdots,k,k+1}(x)=I_{0,1,\cdots,k}(x)+\frac{I_{0,1,\cdots,k-1,k}(x)-I_{0,1,\cdots,k}(x)}{x_{k+1}-x_k}(x-x_k)$$

where $I_i=f(x_i)(i=0,1,\cdots,n)$.

This formula we also call Aitken's successive linear

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ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

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Vol. 8, Issue 4, April 2022

interpolation formula. The advantage of this algorithm is that it is suitable for calculation on a computer and has the feature of automatic node selection and step-by-step comparison of accuracy, and the procedure is simple.

D. Newton interpolation

The Newton interpolation polynomial constructed from Data Table 1 is

$$P_n(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1)\cdots + f[x_0, x_1, \cdots, x_n](x - x_0)(x - x_2)\cdots(x - x_n)$$

When using it for interpolation, we must first calculate the mean difference of each order. And the calculation of each order of mean difference can be reduced to the first order of mean difference one by one. The general k-order mean difference is

$$f[x_0, x_1, \cdots, x_k] = \frac{f[x_1, x_2\cdots, x_k] - f[x_0, x_1, \cdots, x_{k-1}]}{x_k - x_0}$$

The remainder term of the Newton interpolation polynomial is

$$R(x) = f(x) - P_n(x) = f[x, x_0, \cdots, x_n](x - x_0)(x - x_1)\cdots(x - x_n)$$

The nodes of the interpolating polynomials discussed in general are arbitrarily distributed, but in practical applications, if there are many cases of equidistant nodes, the interpolation formula can be further simplified in this case. In the Newtonian mean difference interpolation polynomial in each order mean difference is replaced by the corresponding difference, various forms of equidistant node interpolation formulas are obtained, commonly used is the Newtonian front and back interpolation formulas.

E. Hermite interpolation

Hermite interpolation requires not only that the function values are equal, but also that the derivative values are equal. From the function values and derivative values on the interpolation nodes, as shown in Table 2, an interpolation polynomial of not more than 2n+1 times can be constructed, which is called Hermite interpolation polynomial.

$$H(x) = \sum_{i=0}^{n} [f(x_i)h_i(x) + f'(x_i)l_i(x)]$$

where both $h_i(x)$ and $l_i(x)$ are 2n+1 times interpolated basis functions. The remainder term of the Hermite interpolation is

$$R(x) = f(x) - H(x) = \frac{f^{(2n+1)}(\zeta)}{(2n+2)!}(x - x_0)^2(x - x_1)^2\cdots(x - x_n)^2$$

where $\zeta \in (a, b)$.

F. Segment interpolation

In the whole interpolation interval, as the number of interpolation nodes increases, the number of interpolation polynomials must increase, and high interpolation will produce Runge phenomenon, thus not effectively approximating the interpolated function. It has been proposed to approximate the interpolated function by segmented lower polynomial segments, which is the segmented interpolation method. The method for constructing segmented interpolation polynomials is still the basis function method. That is, the segmented linear interpolation basis functions are first constructed at each interpolation node, and then the basis functions are linearly combined. It has the advantage that the required accuracy can always be obtained as long as the node spacing is sufficiently small, i.e., convergence can always be guaranteed. Another advantage is its local nature, i.e., if a data is modified, the interpolation curve is only affected in a local area. Commonly, segmented linear interpolation and segmented cubic Hermite interpolation are used. But segmented interpolation is less smooth.

G. Triple spline interpolation

The segmented cubic Hermite interpolation we normally use constructs an interpolating polynomial with first-order smoothness overall. However, in practice interpolation will require more smoothness. For example, theoretical models of aircraft shapes, hull release isometrics, etc. often require a second-order smoothness. Commonly used in engineering is the 3 times spline function $S_3(x)$. The basic idea is to divide the interpolation interval n equally and derive the interpolation function $S_3(x)$ by using segmented 3 times Hermite interpolation on each small interval.

(1) $S_3(x)$ is a polynomial $P_i(x)$ not higher than three times on each small interval (i= 0,1,… ,n-1)..

(2) $S_3(x)$ and the interpolated function $f(x)$ coincide at the interpolated node xi, i.e., $S_3(x_i) = f(x_i)$ (i= 0,1,… ,n).

(3) On the whole interval [a, b], $S_3(x)$ has first- and second-order continuous derivatives.

III. APPLICATION OF INTERPOLATION METHOD

The interpolation method has a variety of uses in addition to finding the value of a function.

A. Numerical differentiation methods

Numerical differentiation is the method of finding the derivative value of a function using interpolating polynomials on equidistant nodes. The commonly used two-point and three-point formulas are derived using segmented linear interpolation and segmented parabolic interpolation. It should be noted that these two formulas are only suitable for the derivative values at the nodes. If the derivative is found at other points in the interval it is best to use the spline interpolation function [4].

B. Numerical integration method

For the integral $I = \int_a^b f(x)dx$ if the product function is not clear or its original function is not easy to find, usually according to the data table of $f(x)$ on the integration interval [a, b], construct the interpolation polynomial $P(x)$ instead of $f(x)$, and then derive the integral value.

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

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Vol. 8, Issue 4, April 2022

C. Data Fitting

Data fitting is still the approximation of the expression of the independent and dependent variables by a given set of measured discrete data. In view of the interpolation method whose approximation criterion is that the error at the interpolation point is zero, but considering that in practical applications, sometimes the error at certain specific points is not required to be zero, so as to consider the overall error limit, so that the requested function is not required to pass all the nodes, but the requested approximate function reflects the overall trend of the original function. Data fitting methods are used for this purpose.

IV. Conclusion

We analyzed and discussed the existing interpolation method and some applications of the interpolation method. From our analysis it is clear that the theory of interpolation methods is an effective way to deal with approximate calculations and approximation functions. The application and theory about the interpolation method in other subject areas will be the goal of our further research.

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