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Control Design and Simulation for Leader-Follower AUVs

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***Abstract*—The paper addresses the problem of control design for Autonomous Underwater Vehicle (AUV) formation with control delays. A compact model consisting of the leader motion equation and the followers' motion equations are built. The control-delay system is converted into an equivalent one without delays. Employing optimal control theory for linear systems, the optimal decentralized controls are designed. The simulation validates the proposed decentralized controllers are effective and simple.**

***Index Terms*—AUV, decentralized control, formation, simulation.**

I. Introduction

Up to now, networks has been applied widely, e.g. in field experiments, research projects, commercial products, and military navigations. One useful task using networks is target tracking by Autonomous Underwater Vehicle (AUV) [1], fleets [2], unmanned aerial vehicle [3], multiagent rigid [4] for exploring, formations, and etc. Formation control methods are summarized by [5]. To improve the efficiency of the target hunting, a leader-follower formation algorithm is introduced [6]. A consensus control strategy for multiple unmanned underwater vehicles, a small AUV, with unmeasurable disturbances under the fixed and switching topologies is proposed [7]. While exploiting a leader-follower strategy to formation control and the vector Lyapunov function method to controller design, [8] employed discrete-event approach and supervisory control theory to switch between operational modes. A Lyapunov-based backstepping approach for developing cooperative motion control for multiple AUVs was presented, where the cooperative motion is achieved using a leader–follower formation strategy in the presence of discrete data transmission between the leader AUV and the follower AUVs [9]. A cooperative control problem in three dimensional spaces was considered and finite-time formation for AUVs with constraints on communication range was investigated, where a two-layer finite-time consensus control law is proposed to avoid leading to collapse on formation because of failure leader [10]. However, as is known, network-induced time delays are unavoidable [11]. Our previous work proposed a distributed tracking control algorithm considering coordinative communication among AUVs [12], as shown in Fig. 1. However, the unavoidable network-induced delays were not considered.

In this paper, control-delay between network communication channels is taken into account. A leader and the follower AUV systems are respectively modeled. Combining both leader and follower AUV systems, the leader-follower formation system is established. The exchanging information are assumed to be achieved completely from coordinative communication, where control delay due to network communication is encountered. By employing functional transformation method, the control-delay system is transformed to an equivalent one without time delay. Moreover, the influence generated by control delay is compensated by the integral terms of past control information in designed controller, which can be stored in memories easily. Simulation is conducted through Matlab program with one leader AUV and two follower AUVs. It illustrates that under the designed controllers, two followers track the leader satisfactorily keep prescribed distances eventually. The effectiveness and simplicity of the control algorithm are verified.

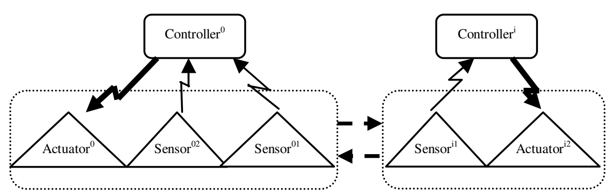

Fig. 1. Leader-follower AUVs communicating over networks.

The paper is organized as follows: after introduction, in Section II, the problem of leader-follower AUV formation with a control delay is formulated. Optimal decentralized controls are proposed in Section III. In Section IV, simulation validation is illustrated. Concluding remarks are made in Section V.

II. Problem Statement

Consider a leader-follower-AUV system. First, the leader-AUV model is given by

$$\dot{x}_0(t) = A_0 x_0(t) + B_0 \gamma_0(t-\tau), \quad y_0(t) = C_0 x_0(t) \tag{1}$$

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where $x_0 \in R^{n_0}$ is the state vector of leader-AUV equation of motion, $\gamma_0 \in R^{m_0}$ is the control input of leader-AUV, and τ is a known constant control delay from controller to actuator. The matrices A_0, B_0, C_0 are constant of leader-AUV equation of motion. Second, the ith $(i = 1, 2, \ldots, k)$ follower-AUV equation of motion is given by

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + B_i \gamma_i(t-\tau), \quad \dot{e}_i(t) = D_i x_0(t) + E_i x_i(t) \\ y_i(t) &= C_i x_i(t), \quad i = 1, 2, \ldots, k \end{aligned} \tag{2}$$

where $x_i \in R^{n_i}$ is the state vector of the ith follower-AUV equation of motion, $\gamma_i \in R^{m_i}$ is the control input of the ith follower-AUV, and τ is a known constant control delay from controller to actuator. The formation error between the leader-AUV and the ith follower-AUV is denoted as $e_i = d_0 - d_i - d_{0i}$, where d_{0i} is the desired nominal distance between them. The matrices A_i, B_i, C_i are constant of the ith follower-AUV equation of motion, and $D_i = \begin{bmatrix} I & 0 \end{bmatrix}$, $E_i = \begin{bmatrix} -I & 0 \end{bmatrix}$ due to the formation error relationship.

The main objective of leader-follower formation is that all the follower-AUVs and the leader-AUV keep a prescribed distance to their neighbors and move in a common direction. In order to design the decentralized control, we introduce the augmented state vector

$$x = \begin{bmatrix} x_0 \\ x_i \\ e_i \end{bmatrix}, \quad y = \begin{bmatrix} y_0 \\ y_i \end{bmatrix} \tag{3}$$

Then, the augmented system is

$$\dot{x}(t) = Ax(t) + \bar{B}\gamma(t-\tau), \quad y(t) = Cx(t) \tag{4}$$

where

$$A = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_i & 0 \\ D_i & E_i & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_0 & 0 \\ 0 & B_i \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_i & 0 \end{bmatrix} \tag{5}$$

The initial condition is $x(0)$. The triple $(A, \bar{B}, C)$ is controllable-observable.

By using the functional transformation

$$z(t) = x(t) + \int_{t-\tau}^{t} \mathrm{e}^{A(t-h)} Bu(h)\mathrm{d}h \tag{6-1}$$

$$\eta(t) = y(t) + C\int_{t-\tau}^{t} \mathrm{e}^{A(t-h)} Bu(h)\mathrm{d}h \tag{6-2}$$

with the initial condition $z(0) = x(0)$, the control-delay system (4) is converted into the equivalent delay-free one, that is,

$$\dot{z}(t) = Az(t) + B\gamma(t), \quad \eta(t) = Cz(t) \tag{7}$$

where $B = \mathrm{e}^{-\tau A}\bar{B}$. It is easily proved that (A, B) is controllable.

Note that, an optimal control for system (7) will not only stabilize the formation error but also minimize the control power consumption. Thus the tradeoff between system states and control input are selected in the performance index. Consider the following average performance index

$$J = \lim_{T\to\infty} \frac{1}{T} \int_0^T \left[z^{\mathrm{T}}(t)Qz(t) + \gamma^{\mathrm{T}}(t)R\gamma(t) \right] \tag{8}$$

where Q is a positive semi-definite matrix, R is a positive definite one, and T is the terminal time. Our purpose is to design the optimal control subject to constrain (7) and performance index (8).

III. Controller Design

According to the Pontryagin Maximum Principle, the optimal control problem consisting of the system (7) with the performance index (8) follows two-point boundary value problem:

$$\dot{z}(t) = Az(t) - BR^{-1}B^{\mathrm{T}}P\lambda(t), \quad \dot{\lambda}(t) = -Qz(t) - A^{\mathrm{T}}\lambda(t) \tag{9}$$

with the two-point values $z(0)$ and $\lambda(\infty) = 0$. The optimized control law is

$$\gamma^*(t) = -R^{-1}B^{\mathrm{T}}\lambda(t) \tag{10}$$

The main result of this paper is demonstrated as follows.

Theorem 1. Consider the leader-follower-AUV system (1) with respect to the performance index (8). There exists the unique optimal control given by

$$\gamma^*(t) = -R^{-1}B^{\mathrm{T}}P\left(x(t) + \int_{t-\tau}^{t} \mathrm{e}^{A(t-h)}Bu(h)\mathrm{d}h \right) \tag{11}$$

where P is the unique positive definite solution of Riccati equation

$$A^{\mathrm{T}}P + PA - PBR^{-1}B^{\mathrm{T}}P + Q = 0 \tag{12}$$

and the minimal performance index is $J_{\min} = z^{\mathrm{T}}(0)Pz(0)$.

Proof. Let the costate vector

$$\lambda(t) = Pz(t) \tag{13}$$

where P is a matrix to be decided. Differentiating both sides of it yields

$$\dot{\lambda}(t) = P\dot{z}(t) \tag{14}$$

Substituting (9-1) into the right side of (14) with (13) yields

$$\dot{\lambda}(t) = (PA - PBR^{-1}B^{\mathrm{T}}P)z(t) \tag{15}$$

Furthermore, substituting (13) into (9-2) yields

$$\dot{\lambda}(t) = -(Q + A^{\mathrm{T}}P)z(t) \tag{16}$$

The identity of Eqs. (15) and (16) produces the Riccati equation (12). Substituting (13) and (6-1) into (10) yields the optimal control (11), where P is the solution of Riccati equation (12). Since the triple (A, B, C) is controllable-observable, according to optimal theory, the solution P of the Riccati equation (12) is unique [13]. Moreover, the closed-loop matrix $A - BR^{-1}B^{\mathrm{T}}P$ is Hurwitz. Therefore, the state $z(t)$ of the closed-loop system (7) under the optimal control $\gamma^*(t) = -R^{-1}B^{\mathrm{T}}Pz(t)$ is exponentially stable. From the relationship (6-1) it can be obviously seen that the state $x(t)$ is exponentially stable under the optimal

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control (11), which implies that $\lim_{t\to\infty} e_i(t) = 0$ $(i = 1, 2, \ldots, k)$ holds for formation errors. The proof of Theorem 1 is completed.

The algorithm of optimal control design for the system (1) is schemed out as follows:

Algorithm 1 Optimal Control Design for system (1):

Step 1: Determine the matrices in (5);

Step 2: Judge the controllability-observability of (A, B, C);

Step 3: Solve the matrix P from Eq. (12);

Step 4: Compute optimal control $u^*(t)$ of (11);

Step 5: Calculate the minimal performance index $J_{\min}$.

IV. Application with AUV

The simulation is conducted by using a leader and two followers of 3DOF AUV model. The state vectors are defined by

$$x_0 = \begin{bmatrix} u_0 \\ v_0 \\ r_0 \\ \theta_0 \\ d_0 \end{bmatrix}, x_1 = \begin{bmatrix} u_1 \\ v_1 \\ r_1 \\ \theta_1 \\ d_1 \end{bmatrix}, x_2 = \begin{bmatrix} u_2 \\ v_2 \\ r_2 \\ \theta_2 \\ d_2 \end{bmatrix}, \gamma_0 = \begin{bmatrix} \gamma_{01} \\ \gamma_{02} \end{bmatrix}, \gamma_1 = \begin{bmatrix} \gamma_{11} \\ \gamma_{12} \end{bmatrix}$$

$$\gamma_2 = \begin{bmatrix} \gamma_{21} \\ \gamma_{22} \end{bmatrix}, e_1 = d_0 - d_1, e_2 = d_0 - d_2$$

and the associated matrices are given by

$$A_0 = A_1 = A_2 = \begin{bmatrix} -0.37 & 0 & 0 & 0 & 0 \\ 0 & -2.78 & -0.63 & 0 & 0 \\ 0 & -5 & -1.97 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1.5 & 0 \end{bmatrix}$$

$$B_0 = B_1 = B_2 = \begin{bmatrix} 0.03 & 0 \\ 0 & 0.28 \\ 0 & -1.60 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_1 = E_2 = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \end{bmatrix}, C_0 = C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The parameter denotations are listed in Table I [12], [14].

TABLE I: Parameter Denotations

Parameter	Denotation
u	forward velocity of AUV in body fixed coordinate (m/sec)
v	lateral velocity of AUV in body fixed coordinate (m/sec)
r	yaw rate (rad/sec)
θ	yaw angle (rad)
d	perpendicular distance from the path (m)
γ_{i1} $(i = 0,1,2$	propeller thrust force (N)
γ_{i2} $(i = 0,1,2$	rudder angle (rad)

Therefore, the augmented system (3)-(5) is performed. Take the control delay $\tau = 0.1\text{s}$, and the weight matrices and the initial states as follows:

$$Q = \mathrm{diag}(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1),\ R = \mathrm{diag}(1,1,1,1,1,1)$$

$$x(0) = (0.2, 0.2, 0.2, 0.2, 0.2, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, \\ -0.1, -0.1, -0.1, -0.1, -0.1, -0.1)$$

The matrix

$$\bar{B}_0 = \bar{B}_1 = \bar{B}_2 = \begin{bmatrix} 0.0311 & 0 \\ 0 & 0.2469 \\ 0 & -1.8015 \\ 0 & 0.1696 \\ 0 & -0.0390 \end{bmatrix}$$

Calculate the gain matrix of the optimal control (11) as

$$-R^{-1}B^{\mathrm{T}}P = \begin{bmatrix} -2.7347 & 0 & 0 & 0 & 0 & 1.3463 & 0 & 0 & 0 & 0 & -0.5774 & 1.3463 & 0 & 0 & 0 & 1 & -0.5774 \\ 0 & -1.4927 & 1.6202 & 3.2100 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.3463 & 0 & 0 & 0 & 0 & -1.9452 & 0 & 0 & 0 & 0 & 0.7887 & 0.5569 & 0 & 0 & 0 & 0 & -0.2113 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1.4927 & 1.6202 & 3.21 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.3463 & 0 & 0 & 0 & 0 & 0.5569 & 0 & 0 & 0 & 0 & -0.2113 & -1.9452 & 0 & 0 & 0 & 0 & 0.7887 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.4927 & 1.6202 & 3.21 & 0 & 0 \end{bmatrix}$$

The simulation is conducted by exploiting Matlab programming. As shown in Fig. 2, the state trajectories of leader and followers converged and the formation error approached to zero. It shows that under the decentralized optimal control (11), the follower-AUVs followed the leader-AUV keeping the prescribed distance and move in a common direction. Moreover, the control-delay is compensated completely by the integral term in the controller. Fig. 3 displays the moving trajectories of the leader and two followers, where the desired distance was set to 10m in x-direction and the followers were separated by 40m to the leader. Although the AUVs were started from random initial positions, they converged to the assigned formation trajectories and maintain mutually the prescribed distances. Even there were control delays between the controller-actuator channels, because of the compensation for control delay in the designed controllers, the formation was still accomplished satisfactorily.

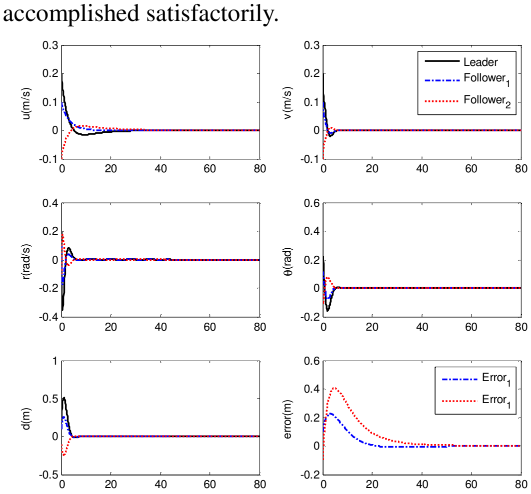

Fig. 2. The state trajectories of the leader and follower AUVs.

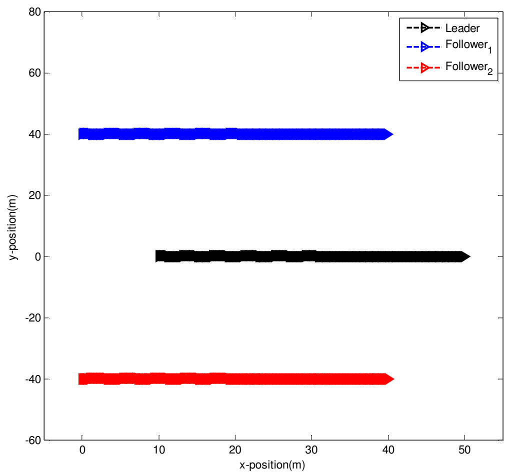

Fig. 3. The leader-follower AUVs moving trajectories.

V. CONCLUSION

In this paper, the optimal control design for decentralized formation control has been proposed. The leader-AUV and follower-AUV systems with control delays are modeled. The augmented system is constructed by defining augmented states. Via a functional transformation, the control-delay system is converted into an equivalent delay-free one. After solving a Riccati equation, the optimal decentralized control is obtained. The influence produced by control delay is compensated by an integral term of past-time controls, which can be easily obtained from computer memories. In simulation, by employing the designed controllers, the follower-AUV follows the leader-AUV satisfactorily after a short regulation period. The effectiveness and the simplicity of the proposed decentralized controllers are validated by simulation.

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