ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

IJARMATE

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE)
Vol. 8, Issue 2,February 2022

Control and Simulation for Chaotic Lorenz Systems

Jia-Qing Song [1], Ji-Yan Wang[2], Yue-Ying Wu[3], Jing Lei [4]

Virtual Experiment Simulation Center, Taishan University, Tai'an 271000, China [1]
School of Mathematics and Statistics, Taishan University, Tai'an 271000, China [2]
School of Mathematics and Statistics, Taishan University, Tai'an 271000, China [3]
School of Mathematics and Statistics, Taishan University, Tai'an 271000, China [4]

***Abstract*—The paper addresses the problem of synchronization control design for chaotic Lorenz systems, where external disturbance and controller-actuator time delay are encountered. To solve the control-delay problem, a functional transformation method is proposed, by using which the control-delay synchronization error system is converted into an equivalent delay-free one, so that the linear part of the controllers can be designed by using traditional methods, e.g. optimal control. As for the external disturbance that the system undertakes, a feedforward term is designed in the controller to compensate for the influence produced by disturbance. The nonlinearity in error system is accurately cancelled by using feedback linearization. After solving a Riccati equation and a Sylvester equation, the nonlinear control consisting of optimal control term, feedforward term, and the feedback linearization term is constructed. The input-to-state stability of the closed-loop system of synchronization error is demonstrated. By using the proposed approach, the nonlinear synchronization controller is designed for chaotic Lorenz system. The effectiveness of the designed controller is validated by simulation.**

***Index Terms*—Chaos synchronization, disturbance, Lorenz, simulation.**

I. Introduction

Chaos synchronization control has been widely applied in circuit [1], Bistatic Radar [2], modem [3], bidirectional communicator [4], semiconductor laser [5], wireless networks [6], etc. Till now, a successful application of chaos synchronization is at secure communications [7]. Synchronization control for chaotic Lorenz systems has been studied with many strategies, such as active feedback control [8], adaptive control [9], sliding model [10], backstepping [11], and observer-based dynamical control [12]. As known to all, time delay occurs frequently over wired and/or wireless networks. Network-induced delays are also unavoidable for chaotic synchronization networks [13]-[15]. Due to the complexity of functional differential equation for time-delay systems, time-delay issue is still open. External disturbance against model enhances the complexity of a chaotic synchronization system with control delay.

This paper investigates into synchronization control design for chaotic Lorenz system, where control delay between controller and actuator channel and the disturbance undertaken by the drive system are considered. To reduce calculation complexity, we transform the input-delay synchronization error system to an equivalent system without delay by using model reduction method [16]-[17]. Hence, the traditional methods for designing controllers can be employed. To compensate for the delay influence to the system, an integral term consisting of past-time control information are added in the control. LQR method is employed to design the linear part control and feedback linearization is adopted to design the nonlinear part so as to accurately cancel the nonlinearity. The input-to-state stability of the closed-loop system of synchronization error is proved according to Lyapunov theory. In the simulation, the proposed method is applied to design the controllers for chaotic Lorenz system. Simulation results show that the designed controllers are effective to guarantee the response system synchronously tracking the drive system, as we as it illustrates that and the proposed method is feasible.

The paper is organized as follows. After an introduction, in Section II, the system description and problem formulation are presented. In section III, the nonlinear control design problem for the synchronization error system is solved. As a consequence, the stability of synchronization error is proved. In order to verify the effectiveness of the proposed approach, numerical simulation is conducted in Section IV. Concluding remarks are given in Section V.

II. System Description

Consider a chaotic synchronization system given by the drive system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + f(x) + Dd(t) \\ x(0) &= x_0 \end{aligned} \tag{1}$$

and the response system

$$\begin{aligned} \dot{x}_s(t) &= Ax_s(t) + B_0u(t-\tau) + f(x_s) \\ x_s(0) &= x_{s0} \end{aligned} \tag{2}$$

where $f: R^n \to R^n$ is a continuous nonlinear functional vector, $x, x_s \in R^n$ are the state vectors, $u \in R^n$ is the control input vector to ensure the response system tracking the drive one, $d \in R$ is an external disturbance that the drive system undertakes, and $\tau > 0$ is a known constant delay between the controller and actuator. The matrices $A \in R^{n\times n}$, $B_0 \in R^{n\times n}$, and $D \in R^n$ are constant and the pair (A, B_0) is controllable.

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

IJARMATE

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE)
Vol. 8, Issue 2,February 2022

There are many chaotic systems, for example, Lorenz attractor, Rössler system, Chua's circuit and Chua's circuit family, which can be described in form of (1). The disturbance is generated from an exosystem

$$\begin{aligned} \dot{w}(t) &= Gw(t) \\ d(t) &= Fw(t) \end{aligned} \tag{3}$$

where $G \in R^{m\times m}$, $F \in R^{1\times m}$, $w \in R^{m}$ is the state vector of disturbance.

Assumption 1. The nonlinear function f satisfies Lipschitz condition with $f(0)=0$.

Remark 1. Since attractors of chaotic systems are bounded, most of chaotic functions assumed satisfying Assumption 1, such as Lorenz, Rössler, Chua's circuit [18].

III. CONTROL DESIGN

Let the synchronization error be $e(t)=x_s(t)-x(t)$. Differentiating both sides of the error equality and substituting the system (1) and (2) into the result, respectively, yield the error equation

$$\begin{aligned} \dot{e}(t) &= Ae(t)+B_0u(t-\tau)+f(x_s)-f(x)-Dd(t) \\ e(0) &= e_0 \end{aligned} \tag{4}$$

The solution of (4) is

$$e(t)=\mathrm{e}^{At}e_0+\int_0^t \mathrm{e}^{A(t-h)}\left[B_0u(h-\tau)+f(x_s)-f(x)-Dd(h)\right]\mathrm{d}h \tag{5}$$

Let $s=h-\tau$. The integral interval becomes $[-\tau, t-\tau]$ and $\mathrm{d}s=\mathrm{d}h$. Replacing $t-\tau$ in (4) by s deduces

$$\begin{aligned} e(t) &= \mathrm{e}^{At}e_0+\int_{-\tau}^{t-\tau}\mathrm{e}^{A(t-s)}\mathrm{e}^{-A\tau}B_0u(s)\mathrm{d}s \\ &\quad+\int_0^t \mathrm{e}^{A(t-h)}\left[f(x_s)-f(x)-Dd(h)\right]\mathrm{d}h \\ &= \mathrm{e}^{At}e_0+\int_0^t \mathrm{e}^{A(t-s)}\mathrm{e}^{-A\tau}B_0u(s)\mathrm{d}s \\ &\quad+\int_0^t \mathrm{e}^{A(t-h)}\left[f(x_s)-f(x)-Dd(h)\right]\mathrm{d}h \\ &\quad-\int_0^{t-\tau}\mathrm{e}^{A(t-s)}\mathrm{e}^{-A\tau}B_0u(s)\mathrm{d}s \end{aligned} \tag{6}$$

with the reason that $u(t)\equiv 0$ when $t\in[-\tau,0)$. Define $B=\mathrm{e}^{-\tau A}B_0$ and a new variable

$$z(t)=e(t)+\int_{t-\tau}^{t}\mathrm{e}^{A(t-s)}Bu(s)\mathrm{d}s \tag{7}$$

From (6), $z(0)=e(0)$. Taking (7) into (6) yields

$$\begin{aligned} z(t) &= \mathrm{e}^{At}z(0) \\ &\quad+\int_0^t \mathrm{e}^{A(t-s)}\left[Bu(s)+f(x_s)-f(x)-Dd(s)\right]\mathrm{d}s \end{aligned} \tag{8}$$

Apparently, (8) is the solution of the differential equation

$$\begin{aligned} \dot{z}(t) &= Az(t)+Bu(t)+\Phi(x_s,x)-Dd(t) \\ z(0) &= z_0 \end{aligned} \tag{9}$$

where

$$\Phi(x_s,x)\triangleq\begin{bmatrix}\psi_1(x_s,x)\\ \psi_2(x_s,x)\\ \psi_3(x_s,x)\end{bmatrix}=f(x_s)-f(x)$$

The delay-free system (9) is equivalent to the control-delay system (4) [16]-[17]. Therefore, the control-delay system (4) is converted into its equivalent delay-free on (9). Now, our aim is to design a controller to guarantee the stability of synchronization error.

Theorem 1. Consider the chaotic drive system (1) and the response system (2). Then, under the nonlinear control

$$u(t)=-B^{\mathrm{T}}Pz(t)-B^{\mathrm{T}}P_1d(t)-B^{-1}\Phi(x_s,x) \tag{10}$$

where P is positive definite and unique solution of the following Riccati equation

$$A^{\mathrm{T}}P+PA-PBB^{\mathrm{T}}P+Q=0 \tag{11}$$

P_1 is the solution of the following Sylvester equation

$$(A-BB^{\mathrm{T}}P)^{\mathrm{T}}P_1+P_1G-PDF=0 \tag{12}$$

Q is a positive semi-definite matric, the synchronization errors is asymptotically stable.

Proof. Denote the control (10) as

$$u(t)=\tilde{u}(t)-B^{-1}\Phi(x_s,x) \tag{13}$$

where $\tilde{u}(t)=-B^{\mathrm{T}}Pz(t)-B^{\mathrm{T}}P_1d(t)$. Then, substituting (13) into the error system (9) yields the closed-loop error system

$$\begin{aligned} \dot{z}(t) &= Az(t)+B\left(\tilde{u}(t)-B^{-1}\Phi(x_s,x)\right)+\Phi(x_s,x)-Dd(t) \\ &= Az(t)+B\tilde{u}(t)-BB^{-1}\Phi(x_s,x)+\Phi(x_s,x)-Dd(t) \\ &= Az(t)+B\tilde{u}(t)-Dd(t) \end{aligned} \tag{14}$$

According to Maximum Value Principle, the LQR problem of system (14) leads to the following two-point boundary value problem

$$\dot{z}(t)=Az(t)-BB^{\mathrm{T}}\lambda(t)-Dd(t) \tag{15-1}$$

$$\dot{\lambda}(t)=-Qz(t)-A^{\mathrm{T}}\lambda(t) \tag{15-2}$$

with the boundary values $z(0)=z_0$ and $\lambda(\infty)=0$, where

$$\lambda(t)=Pz(t)+P_1w(t) \tag{16}$$

is the costate vector, Q is the weight matrix from the performance index, and the optimal control is $\tilde{u}(t)=-B^{\mathrm{T}}Pz(t)-B^{\mathrm{T}}P_1w(t)$ in which P and P_1 are to be specified. Differentiating both sides of (16) and substituting (15-1) and (3) into the result yield

$$\dot{\lambda}(t)=(PA-PBB^{\mathrm{T}}P)z(t)-(PDF-P_1G+PBB^{\mathrm{T}}P_1)w(t) \tag{17}$$

Substituting (16) into (15-2) yields

$$\dot{\lambda}(t)=-(Q+A^{\mathrm{T}}P)z(t)-A^{\mathrm{T}}P_1w(t) \tag{18}$$

The eventual identity of the equalities (17) and (18) results in the Riccati equation (11) and the Sylvester equation (12), from which the matrices P and P_1 are now solvable. According to LQR theory, the solution P of the Riccati equation (11) is unique and the closed-loop matrix $A-BB^{\mathrm{T}}P$ is Hurwitz. Therefore, the optimal control

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

IJARMATE

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE)
Vol. 8, Issue 2,February 2022

$u(t) = -B^{\mathrm{T}}Pz(t) - B^{\mathrm{T}}P_1 w(t)$ is obtained and so does the nonlinear control (10). Now, we check the stability property of the closed-loop system (9) under (10).

Denote a Lyapunov function candidate $V(z) = z^{\mathrm{T}}Pz$. The derivative of the Lyapunov function along the closed-loop system (9) under (10) deduces

$$\begin{aligned}\dot{V} &= \dot{z}^{\mathrm{T}}Pz + z^{\mathrm{T}}P\dot{z} \\ &= z^{\mathrm{T}}\left[(A-BB^{\mathrm{T}}P)^{\mathrm{T}} + P(A-BB^{\mathrm{T}}P)\right]z \\ &\quad -2w^{\mathrm{T}}(P_1^{\mathrm{T}}BB^{\mathrm{T}}P + F^{\mathrm{T}}D^{\mathrm{T}}P)z\end{aligned} \tag{19}$$

Since $A - BB^{\mathrm{T}}P$ is Hurwitz, from Lyapunov theory, there is a symmetric matrix Q_1 that satisfies

$$(A-BB^{\mathrm{T}}P)^{\mathrm{T}} + P(A-BB^{\mathrm{T}}P) = -Q_1 .$$

Meanwhile, from (12), it yields

$$P_1^{\mathrm{T}}BB^{\mathrm{T}}P + F^{\mathrm{T}}D^{\mathrm{T}}P = P_1^{\mathrm{T}}A + G^{\mathrm{T}}P_1^{\mathrm{T}}$$

Hence, the (19) deduces

$$\begin{aligned}\dot{V} &= -z^{\mathrm{T}}Q_1 z - 2w^{\mathrm{T}}(P_1^{\mathrm{T}}A + G^{\mathrm{T}}P_1^{\mathrm{T}})z \\ &\le -\tfrac{1}{2}\|Q_1\|\|z\|^2 - \left[\tfrac{1}{2}\|Q_1\|\|z\| - 2\left\|P_1^{\mathrm{T}}A + G^{\mathrm{T}}P_1^{\mathrm{T}}\right\|\|w\|\right]\|z\| \\ &\le -\tfrac{1}{2}\|Q_1\|\|z\|^2\end{aligned}$$

for $\|z\| \ge 4\left\|P_1^{\mathrm{T}}A + G^{\mathrm{T}}P_1^{\mathrm{T}}\right\|\|w\|/\|Q_1\|$. According to control theory, the closed-loop system of (9) under (10) is input-to-state stable, which implies that whether the closed-loop of (9) is exponential stable or ultimate bounded depends on the property of disturbance $d(t)$. In other words, when the disturbance is periodical signal, the closed-loop system of (9) under control (10) is input-to-state stable. And when the disturbance is attenuated signal, the closed-loop system of (9) under control (10) is exponentially stable. Namely, as $t \to \infty$, $z(t) \to 0$. From (7), apparently, $e(t) \to 0$ as $t \to \infty$. The proof is completed.

Now we apply the abovementioned procedure into the chaotic Lorenz system, given by

$$\begin{aligned}\dot{x}_1(t) &= -ax_1(t) + ax_2(t) + d(t) \\ \dot{x}_2(t) &= cx_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) &= -bx_3(t) + x_1(t)x_2(t)\end{aligned} \tag{20}$$

with $a = 10, b = 8/3, c = 28$, where there is a chaotic attractor. The controlled response system is given by

$$\begin{aligned}\dot{x}_{s1}(t) &= -ax_{s1}(t) + ax_{s2}(t) + u_1(t-\tau) \\ \dot{x}_{s2}(t) &= cx_{s1}(t) - x_{s2}(t) - x_{s1}(t)x_{s3}(t) + u_2(t-\tau) \\ \dot{x}_{s3}(t) &= -bx_{s3}(t) + x_{s1}(t)x_{s2}(t) + u_3(t-\tau)\end{aligned} \tag{21}$$

Subtracting (21) from (20) yields the synchronization error system

$$\begin{aligned}\dot{e}_1(t) &= -ae_1(t) + ae_2(t) + u_1(t-\tau) \\ \dot{e}_2(t) &= ce_1(t) - e_2(t) - x_{s1}(t)x_{s3}(t) + x_1(t)x_3(t) + u_2(t-\tau) \\ \dot{e}_3(t) &= -be_3(t) + x_{s1}(t)x_{s2}(t) - x_1(t)x_2(t) + u_3(t-\tau)\end{aligned} \tag{22}$$

It can be observed that the system (22) is the system (4) with the matrices and the vectors as follows

$$A = \begin{bmatrix} -a & a & 0 \\ c & -1 & 0 \\ & 0 & -b \end{bmatrix}, \quad B_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad \Phi(x_s, x) = \begin{bmatrix} 0 \\ -x_{s1}(t)x_{s3}(t) + x_1(t)x_3(t) \\ x_{s1}(t)x_{s2}(t) - x_1(t)x_2(t) \end{bmatrix}$$

Notice that there is a known constant control delay τ existing between the controller and actuator. Through the functional transformation (7), the control-delay system (22) is converted into the equivalent delay-free one

$$\begin{aligned}\dot{z}_1(t) &= -az_1(t) + az_2(t) + b_{11}u_1(t) + b_{12}u_2(t) + b_{13}u_3(t) \\ \dot{z}_2(t) &= cz_1(t) - z_2(t) - x_{s1}(t)x_{s3}(t) + x_1(t)x_3(t) + b_{21}u_1(t) \\ &\quad + b_{22}u_2(t) + b_{23}u_3(t) \\ \dot{z}_3(t) &= -bz_3(t) + x_{s1}(t)x_{s2}(t) - x_1(t)x_2(t) + b_{31}u_1(t) \\ &\quad + b_{32}u_2(t) + b_{33}u_3(t)\end{aligned} \tag{23}$$

where

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Notice that the matrix B depends on the value of control delay τ. Let the control

$$\begin{aligned}u(t) &= -B^{\mathrm{T}}P\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} - B^{\mathrm{T}}P_1 w(t) \\ &\quad - \begin{bmatrix} 0 \\ x_{s1}(t)x_{s3}(t) - x_1(t)x_3(t) \\ -x_{s1}(t)x_{s2}(t) + x_1(t)x_2(t) \end{bmatrix}\end{aligned} \tag{24}$$

in which P is solved from the Riccati equation (11) taking $Q = I$ and P_1 is solved from the Sylvester equation (12). Therefore, according to Theorem 1, the exponential stability of the synchronization error is guaranteed.

IV. SIMULATION

In simulation, we will verify the correctness of the designed controllers and the effectiveness of the proposed approach.

Take the parameter values as $a = 10, b = 8/3, c = 28$ in Lorenz system (20). The Lorenz chaos are illustrated in Fig. 1.

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE)
Vol. 8, Issue 2,February 2022

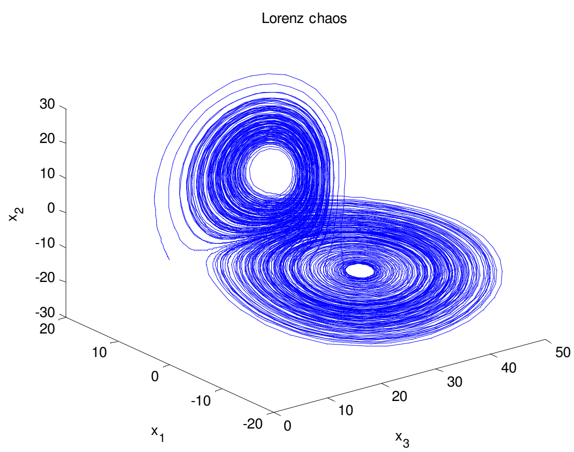

Fig. 1. The chaotic Lorenz.

Take the control delay $\tau = 0.01$s in the synchronization system (21). Then, we get the synchronization error system (22). Transform the control-delay system (22) into delay-free one (23). The controllers is designed as (24), where the matrices values are as follows:

$$B = \begin{bmatrix} 1.1202 & -0.1062 & 0 \\ -0.2973 & 1.0247 & 0 \\ 0 & 0 & 1.0270 \end{bmatrix}$$

$$P = \begin{bmatrix} 18.6785 & 14.5585 & 0 \\ 14.5585 & 11.3825 & 0 \\ 0 & 0 & 0.1810 \end{bmatrix}$$

$$P_1 = \begin{bmatrix} -1.4494 & -1.3401 & 0 \\ -1.1291 & -1.0454 & 0 \\ 0 & 0 & -0.0974 \end{bmatrix}$$

Thus, the controller (24) is

$$u(t) = -\begin{bmatrix} 16.5956 & 12.9246 & 0 \\ 12.9342 & 10.1173 & 0 \\ 0 & 0 & 0.1859 \end{bmatrix}\begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix} - \begin{bmatrix} -1.288 & -1.1904 & 0 \\ -1.003 & -0.9289 & 0 \\ 0 & 0 & -0.1001 \end{bmatrix}\begin{bmatrix} w_1(t) \\ w_2(t) \\ w_3(t) \end{bmatrix} - \begin{bmatrix} 0 \\ x_{s1}(t)x_{s3}(t) - x_1(t)x_3(t) \\ -x_{s1}(t)x_{s2}(t) + x_1(t)x_2(t) \end{bmatrix}$$

We perform the simulation with an attenuated disturbance $w_1(t) = \exp(-t)$ and $w_2(t) = w_3(t) = 0$ with $G = \text{diag}(-1,1,1)$ and $F = D = I_3$. The simulation is conducted in Matlab. The initial states are set as

$$\left(x_1(0), x_2(0), x_3(0), x_{s1}(0), x_{s2}(0), x_{s3}(0)\right) = (0, 0.1, 0.1, 0, 0, 0.1)$$

The synchronized states of chaotic Lorenz system are shown in Fig. 2. The synchronization errors are shown in Fig. 3, which are observed convergent to zeroes at about 2sec. It shows that the designed controllers ensure the response system tracks the Lorenz system synchronously.

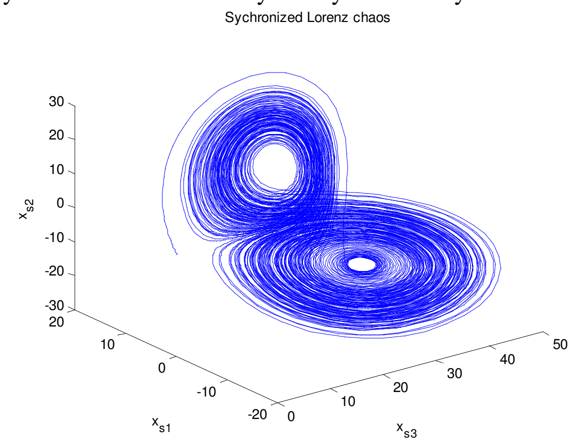

Fig. 2. Synchronized chaotic Lorenz.

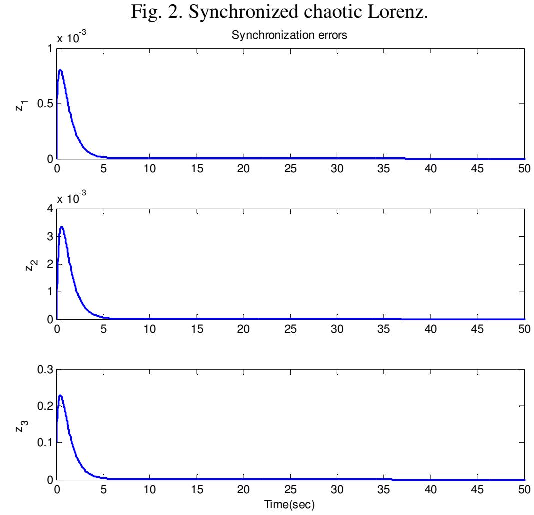

Fig. 3. Synchronization errors.

V. Conclusion

The paper designs the synchronization controllers for chaotic Lorenz systems in the presence of a control delay and an external disturbance. It presented a functional transformation method so that the control-delay synchronization error system was converted into an equivalent delay-free system. The control was easily designed, which was combined by a linear part using LQR method, a feedforward term compensating for the influence produced by disturbance, and an accurate nonlinear term cancelling the nonlinearity exactly. The effectiveness and feasibility of the proposed method and the controllers are validated by the simulation.

Acknowledgment

This work was supported by the Teaching Reform and Research Project of Taishan University in 2021: Research on Constructing the Information-Based Teaching Mode and

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE)
Vol. 8, Issue 2,February 2022

Course System of the Information and Computing Science Speciality.

REFERENCES

[1] K.M. Cuomo, A.V. Oppenheim, and S.H. Strogatz, "Synchronization of Lorenz-based chaotic circuits with applications to communications," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 40, no. 10, pp. 626-633, 1993.

[2] C.S. Pappu, B.C. Flores, and J.E. Boehm, "An electronic implementation of Lorenz chaotic oscillator synchronization for bistatic radar applications," I*EEE Transactions on Aerospace and Electronic Systems*, vol. 53, pp. 2001-2013, 2017.

[3] R. Carrasco and C. Pointon, "Modem-based communications using synchronized chaos," I*nternational Journal of Communication Systems*, vol. 14, pp. 305-317, 2001.

[4] G. Kaddoum, "Wireless chaos-based communication systems: a comprehensive Survey," *IEEE Access*, vol. 4, pp. 2621-2648, 2016.

[5] J. Ohtsubo, "Chaos synchronization and chaotic signal masking in semiconductor lasers with optical feedback," *IEEE Journal of Quantum Electronics*, vol. 38, no. 9, pp. 1141-1154, 2002.

[6] R. Nunez, "An optimal chaotic bidirectional communicator for hidden information, based on synchronized Lorenz circuits," *Journal of Applied Research and Technology*, vol. 2, pp. 5-20, 2004.

[7] S. Banerjee, *Chaos synchronization and cryptogrphy for secura communications: Applications for encryption*, IGI Global, 2010.

[8] X. Wang and J. Song, "Synchronization of the fractional order hyperchaos Lorenz systems with activation feedback control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 8, pp. 3351-3357, 2009.

[9] T. Liao and S. Lin, "Adaptive control and synchronization of Lorenz systems," *Journal of The Franklin Institute-engineering and Applied Mathematics*, vol. 336, no. 6, pp. 925-937, 1999.

[10] S. Vaidyanathan and S. Sampath, "Global chaos synchronization of hyperchaotic Lorenz systems by sliding mode control," *International Conference on Digital Image Processing*, pp.156-164, 2011.

[11] X. Tan, J. Zhang, and Y. Yang, "Synchronizing chaotic systems using backstepping design," *Chaos Solitons & Fractals*, vol. 16, no. 1, pp. 37-45, 2003.

[12] Z. Zhang, H. Shao, Z. Wang, and H. Shen, "Reduced-order observer design for the synchronization of the generalized Lorenz chaotic systems," *Applied Mathematics and Computation*, vol. 218, no. 14, pp. 7614-7621, 2012.

[13] L. Li, H. Peng, Y. Yang, and X. Wang, "On the chaotic synchronization of Lorenz systems with time-varying lags," *Chaos Solitons & Fractals*, vol. 41, no. 2, pp. 783-794, 2009.

[14] H. Chen, R. Chen, and M. Ji, "Finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system," *Journal of Information and Computing Science*, vol. 10, no. 4, pp. 265-270, 2015.

[15] H. Wang, J.-M. Ye, Z.-H. Miao, and E.A Jonckheere, "Robust finite-time chaos synchronization of time-delay chaotic systems and its application in secure communication," *Transactions of the Institute of Measurement and Control*, vol. 40, no. 4, pp. 1082–1091, 2016.

[16] Z. Artstein, "Linear systems with delayed controls: a reduction, IEEE Transactions on Automatic Control," vol. AC-27, no. 4, pp. 869-879, 1982.

[17] Jing Lei, *Optimal vibration control for suspension systems with time-delays*, Kunming: Yunnan University Press, 2013, in Chinese.

[18] J.G. Lu, X.F. Wang, and Z.Q. Wang, "State feedback approach to controlling and synchronizing continuous-time chaotic systems," *Control and Decision*, vol. 16, no. 4, pp. 476-479, 2001.

Jia-Qing Song received the Bachelor from Shanghai University of International Business and Economics in 1991. He is an experimenter in Taishan University. His research interests include simulation experiment. Email: jiaqing_song@126.com.

Ji-Yan Wang received her Bachelor from Taishan University in 2010. She is an assistant experimentalist at Taishan University. Her research interests include computer technology and application. Email: wangjiyan1973@163.com.

Yue-Ying Wu received her B.S. from Taishan University in 2010 and her M.S. from Qingdao University of Science and Technology in 2018. She is an assistant at Taishan University. Her research interests include computer technology and application. Email: wyy7758525@126.com.

Jing Lei received the B.S., M.S., and Ph.D. degrees in Information Science and Engineering from Ocean University of China, in 2003, 2007, and 2010, respectively. She is a professor at Taishan University. Her research interests include control theory and application, computer application technology. Email: elizabethia@126.com.