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# Ruin Probability of Risk Model with Interference under Stochastic Interest Rate

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***Abstract*—In this paper, we discuss the ruin probability of risk model with interference term under stochastic interest rate on the basis of classical risk model. And the properties of surplus process are discussed. The expression of ruin probability is given by using the properties of surplus process.The ruin probability considering interest rate and inflation is obtained.**

**Keywords—Surplus process; Random interest rate; Wiener process; Interference; ruin probability.**

## I. Introduction

In the general risk theoretical model, the interest rate is set as a constant, but many economic behaviors are long-term. During this period, government policies, economic cycles and other factors will cause uncertainty, that is, interest rate volatility, which means the randomness of interest rate. Moreover, it is recognized that the risk generated by the randomness of interest rates (for insurance companies) may be quite large and cannot be dispersed by increasing the sales volume of policies. However, the risks generated by the randomness of accidents (for insurance companies) can be dispersed by selling a large number of policies. In this sense, interest rate risk should be paid more attention than the risk of accidents. So this paper studies the ruin probability model under stochastic interest rate. Moreover, because insurance companies have uncertain income and payment in their daily operations, random interference terms are added to the model.

The surplus process of the insurance company is

$$R(t)=(x+ct)(1+I)-S(t)+\sigma W(t)$$

Among：

$R(t)$ ：Surplus of the insurance company at the moment t;

$x=R(0)$ :The initial surplus of the insurance company;

$c$ ：Premium collected in unit time;

$I$ ：Random interest rate and $E(I)=i$ ， $Var(I)=\theta^2$

;

$S(t)$ ：Prime Minister's compensation amount up to the moment t, and

$$S(t)=\sum_{k=1}^{N(t)} X_k .$$

Among： $X_1, X_2, \Lambda$ independent of each other and distributed in the same distribution function $F(\cdot)$ ；

$\{N(t), t\geq 0\}$ is the Possion of the parameter $\mu$ ,and $X_1, X_2, \Lambda$ is independent of each other.

$W(t)$ ：Standard Wiener process,represents the uncertain earnings and payments of the insurance company. $\sigma>0$ .

Arder

$$T=\inf\{t \mid R(t)<0\}$$

indicates the moment of bankruptcy ，and

$$\psi(x)=P(T<\infty)$$

is probability of bankruptcy.

In this paper, we discuss the properties of surplus process, and give the expression of ruin probability by using the properties of surplus process.

## II. Nature of Surplus Process

**Nature 1** Surplus process $\{R(t), t\geq 0\}$ is stable independent increment.

**Prove** $t_0<t_1<\Lambda<t_n$ ,We can get

$$R(t_i)-R(t_{i-1})=$$
$$c(1+I)(t_i-t_{i-1})-[S(t_i)-S(t_{i-1})]+\sigma[W(t_i)-W(t_{i-1})] .$$

So

$1+I, S(t_1)-S(t_0), S(t_2)-S(t_1), \Lambda$ ,
$S(t_n)-S(t_{n-1}), W(t_1)-W(t_0)$, $W(t_2)-W(t_1), \Lambda$ ,
$W(t_n)-W(t_{n-1})$ are mutually independent. Surplus process $\{R(t), t\geq 0\}$ has independent increments.

Because

$R(t+s)-R(t)=cs(1+I)-[S(t+s)-S(t)]+\sigma[W(t+s)-W(t)]$ ， $cs(1+I)$ ， $S(t+s)-S(t)$ ， $W(t+s)-W(t)$ have the same distribution respectively for every $t\geq 0$ .So, $R(t+s)-R(t)$ has the same distributionfor every $t\geq 0$ , $\{R(t), t\geq 0\}$ has smooth increments.

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So,surplus process $\{R(t), t \geq 0\}$ has smooth increments.

**Nature2** $E[R(t)] = (x+ct)(1+i) - \mu p_1 t$ ,
$Var[R(t)] = (x+ct)^2\theta^2 + \mu\, p_2 t + \sigma^2 t$ .
Among $p_k = \int x^k dF(x), k = 1,2$.

**Prove** For $t$

$$E[R(t)] = E[(x+ct)(1+I) - S(t) + \sigma W(t)] .$$
$$= (x+ct)[1+E(I)] - E[S(t)] + \sigma E[W(t)]$$

Because

$$E(I) = i\ ,$$
$$E[W(t)] = 0\ ,$$
$$E[S(t)] = E\{E[S(t)|N(t)]\} = E[p_1 N(t)] = p_1 E[N(t)] = \mu p_1 t\ .$$

So

$$E[R(t)] = (x+ct)(1+i) - \mu p_1 t\ .$$

Because of the random rocess $I$ , $\{S(t), t \geq 0\}$, $\{W(t), t \geq 0\}$ are mutually independent,so

$$Var[R(t)] = (x+ct)^2 Var(1+I) + Var[S(t)] + \sigma^2 Var[W(t)]\ .$$
$$Var[W(t)] = t\ ,$$
$$Var(1+I) = \theta^2\ ,$$
$$Var[S(t)] = E\{Var[S(t)|N(t)]\} + Var\{E[S(t)|N(t)]\} = \mu p_2 t\ ,$$

So

$$Var[R(t)] = (x+ct)^2\theta^2 + \mu\, p_2 t + \sigma^2 t\ .$$

In order to ensure the stable operation of the insurance company, the insurance premium income should be greater than the claim amount in unit time. According to nature 2, it is necessary to assume that $c(1+i) > \mu p_1$ .

## III. RUIN PROBABILITY

**Definition** $M_X(r)$ is the moment generating function of individual claim amount. $R$ is nonzero positive solution of the equation

$$-rc(1+i) + \mu[M_X(r) - 1] + \tfrac{1}{2}\sigma^2 r^2 = 0 \qquad (1)$$

$R$ is called adjustment coefficient,the equation(1) is called adjustment coefficient equation.

**lemma** The solution of the adjustment coefficient equation (1).

**Prove** Let

$g(r) = -rc(1+i) + \mu[M_X(r) - 1] + \frac{1}{2}\sigma^2 r^2 = 0$ ，则

$$\frac{dg(r)}{dr} = -c(1+i) + \mu E(Xe^{rX}) + \sigma^2 r\ ,$$

$$\frac{d^2 g(r)}{dr^2} = \mu E(X^2 e^{rX}) + \sigma^2 > 0\ .$$

So $g(r)$ is a lower convex function.The equation (1) is at most two solutions,and $g(0) = 0$, $r = 0$ is a trivial solution. Because

$$\left.\frac{dg(r)}{dr}\right|_{r=0} = -c(1+i) + \mu p_1 < 0\ .$$

And $r \to +\infty$ , $g(r) \to +\infty$ ,So the positive solution $R$ is unique.

The following theorem gives the general expression of ruin probability.

**Theorem1** $\psi(x) = \dfrac{E[e^{-Rx(1+I)}]}{E[e^{-R\cdot R(T)+RcT(I-i)}|T<\infty]}$ .

$R$ is the adjustment coefficient.

**Prove** $t > 0$ , $r > 0$ ,we get

$$E[e^{-r\cdot R(t)}] = E[e^{-r\cdot R(t)}|T \leq t]P(T \leq t) + E[e^{-r\cdot R(t)}|T > t]P(T > t) \qquad (2)$$

Because $R(t) = (x+ct)(1+I) - S(t) + \sigma W(t)$ ,

$$E[e^{-r\cdot R(t)}] = E[\exp[-rx(1+I) - rct(1+I) + rS(t) - r\sigma W(t)] =$$
$$E\{\exp[-rx(1+I) - rct(1+I) + \mu t[M_X(r) - 1] + \tfrac{1}{2}\sigma^2 r^2 t]\}\ . \qquad (3)$$

Because

$R(t) = R(T) + [R(t) - R(T)] =$
$R(T) + c(1+I)(t-T) - [S(t) - S(T)] + \sigma[W(t) - W(T)]$ .

For $T$ , $S(t)-S(T)$ , $W(t)-W(T)$ , $R(T)$ and $I$ are mutually independent. $S(t)-S(T)$ is the compound Possion distribution of $\mu(t-T)$ . $W(t)-W(T) \sim N(0, t-T)$ .

So

$$E[e^{-r\cdot R(t)}|T \leq t]P(T \leq t) =$$
$$E\{\exp[-rR(T) - rc(1+I)(t-T) + \mu(t-T)(M_X(r) - 1)$$
$$+\tfrac{1}{2}\sigma^2 r^2 t]|T \leq t\}\ P(T \leq t)\ . \qquad (4)$$

So

$$-Rc(1+i) + \mu[M_X(R) - 1] + \tfrac{1}{2}\sigma^2 R^2 = 0\ .$$

(3) become

$$E[e^{-R\cdot R(t)}] = E\{\exp[-Rx(1+I) - Rct(I-i)]\}\ .$$

(4) become

$$E[e^{-R\cdot R(t)}|T \leq t]P(T \leq t) =$$
$$E\{\exp[-RR(T) - Rc(t-T)(I-i)]|T \leq t\}\ P(T \leq t) =$$
$$E\{\exp[-Rct(I-i)]\}E\{\exp[-RR(T) + RcT(I-i)]|T \leq t\}P(T \leq t)$$
$$E\{\exp[-RR(T) + RcT(I-i)]|T \leq t\}P(T \leq t) \to$$
$$E\{\exp[-RR(T) + RcT(I-i)]|T \leq \infty\}\psi(x)\ \ (t \to \infty)\ .$$

For

$$E[e^{-r\cdot R(t)}|T > t]P(T > t)\ . \qquad (5)$$

Because

$E[R(t)] = (x+ct)(1+i) - \mu p_1 t$ ,
$Var[R(t)] = (x+ct)^2\theta^2 + \mu\, p_2 t + \sigma^2 t$ .

Let $\alpha = c(1+i) - \mu p_1 > 0$ ,so

$$q(t) = x(1+i) + \alpha t - [(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{\frac{1}{3}}.$$

And let $t \to \infty$ ,we get $q(t) > 0$。

So, ( 5 ) become

$$E[e^{-R\cdot R(t)} | T > t, 0 \le R(t) \le q(t)] P(T > t, 0 \le R(t) \le q(t)) +$$
$$E[e^{-R\cdot R(t)} | T > t, R(t) > q(t)] P(T > t, R(t) > q(t))$$
$$\le P(R(t) \le q(t)) + e^{-R\cdot q(t)}.$$

Applying Chebyshev inequality, we get

$$P(R(t) \le q(t))$$
$$= P(R(t) \le x(1+i) + \alpha t - [(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{\frac{1}{3}}) \le$$
$$P(|R(t) - (x(1+i) + \alpha t)| \ge [(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{\frac{1}{3}}) \le$$
$$Var(R(t))[(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{-\frac{2}{3}} =$$
$$[(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{\frac{1}{3}}.$$

So

$$E[e^{-R\cdot R(t)}] = E\{\exp[-Rx(1+I) - Rct(I-i)]\} \le$$
$$E\{\exp[-RR(T) - Rc(t-T)(I-i) | T < t]\} P(T \le t)$$
$$+ [(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{\frac{1}{3}} + e^{-R\cdot q(t)}.$$

So

$$E\{\exp[-Rx(1+I)]\} \le E\{\exp[-RR(T)$$
$$+ RcT(I-i)] | T < t\} P(T \le t) +$$
$$\frac{[(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{\frac{1}{3}} + e^{-R\cdot q(t)}}{E[e^{-Rct(I-i)}]}.$$

Let $t \to \infty$ ,we get $P(T \le t) \to \psi(u)$ , $e^{-R\cdot q(t)} \to 0$ ,and

$$E[e^{-Rct(I-i)}] = \int_0^{+\infty} e^{-Rct(x-i)} p(x)dx. \quad (6)$$

So ( 6 ) become

$$E[e^{-Rct(I-i)}] = \int_0^{+\infty} e^{-Rct(x-i)} p(x)dx >$$
$$\int_{x_0-\delta}^{x_0+\delta} e^{-Rct(x-i)} p(x)dx > 2\delta d e^{-Rct(x_0+\delta-i)} \to \infty (t \to \infty).$$

Let $t \to \infty$ ,we get

$$\frac{[(x+ct)^2\theta^2 + \mu p_2 t + \sigma^2 t]^{\frac{1}{3}} + e^{-R\cdot q(t)}}{E[e^{-Rct(I-i)}]} \to 0.$$

and

$$E[e^{-Rx(1+I)}] = E[e^{-R\cdot R(T) + RcT(I-i)} | T < \infty] \psi(x).$$

即

$$\psi(x) = \frac{E[e^{-Rx(1+I)}]}{E[e^{-R\cdot R(T) + RcT(I-i)} | T < \infty]}.$$

**Inference 1** If $I = 0$ ,we get

$$\psi(x) = \frac{e^{-Rx}}{E[e^{-R\cdot R(T)} | T < \infty]}.$$

It is the ruin probability of the model with interference risk without considering the interest rate factor. [6] discussed that Liver tumor division in restorative pictures has been generally considered as of late, of which the Level set models show an uncommon potential with the advantage of overall optima and functional effectiveness. The Gaussian mixture model (GMM) and Expected Maximization for liver tumor division are introduced. [7] discussed that In surgical planning and cancer treatment, it is crucial to segment and measure a liver tumor's volume accurately. Because it would involve automation, standardisation, and the incorporation of complete volumetric information, accurate automatic liver tumor segmentation would substantially affect the processes for therapy planning and follow-up reporting.

**Inference 2** If $I = i$ , $I$ is constant interest rate,we get

$$\psi(x) = \frac{E[e^{-Rx(1+i)}]}{E[e^{-R\cdot R(T)} | T < \infty]}.$$

That is the ruin probability of the risk model with interference under constant interest rate.

## IV. Model Considering Interest Rate and Inflation

If $J$ is the inflation rate , Surplus process is

$$R(t) = (x+ct)(1+I-J) - S(t) + \sigma W(t).$$

We get the theorem 2.

**Theorem 2** $\psi(x) = \dfrac{E[e^{-Rx(1+I-J)}]}{E\{e^{-R\cdot R(T) + RcT[(I-i)+(J-j)]} | T < \infty\}}$.

$R$ is the adjustment coefficient.

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