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Ruin Probability of the Generalized Compound Binomial Risk Model by Diffusion

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Abstract-In this paper, we discuss the generalized compound binomial risk model by diffusion. Then we discuss the characters of surplus process. And we get two theory of the ruin probability in this new model

Keywords-Diffusion, ruin probability, surplus process

I. INTRODUCTION

It deals with the stochastic risk in the affairs of the insurance mainly in classical risk theory, discusses such questions as the existence probability within limited time ,etc.. The model is divided into continuous time model and discrete time model according to time mainly. In recent years, domestic and international research of the model very many to continuous time, and get a lot of kind results too, but study on discrete model is less and mostly concentrate on the studying of double binomial models of totally dispersed complex. And it has many factors not confirmed in managing ordinary things in the insurance company, there are many income and payment not confirmed.

We now first introduce some basic results.

Definition1.1 Set

$$R(t) = u + ct - S(t) + aW(t) \quad , \quad S(t) = \sum_{i=1}^{N(t)} Z_i, t \geq 0$$

where u denotes the initial capital; c is the premium income; $\{N(t), t \geq 0\}$ is a Poisson process with parameter $\lambda > 0$,it counts the number of the claim in the interval $(0, t)$ is a non-negative sequence of i. i. d. random variables, Z_i denotes the aggregate claims between times 0 and t . $R(t)$ is the surplus of an insurance company at times t ; $\{N(t), t \geq 0\}$ and Z_i are independent each other. $\{W(t), t \geq 0\}$ is a standard Brownian motion which stands the not confirmed income of the insurance company .The model is called classical risk process by diffusion [1]

Definition1.2 Set

$$R(t) = u + cM(t) - S(t) + aW(t) \quad S(t) = \sum_{i=1}^{N(t)} Z_i, t \geq 0$$

where $\{M(t), t \geq 0\}$ is a Poisson stochastic variable with parameter μ .The other variables have the same definition as definition1.1. The model is called ruin probability in risk with two Poisson processes by diffusion [2]

Definition1.3 Set

$$R(t) = u + \sum_{i=1}^{N_1(t)} x_i - \sum_{i=1}^{N_2(t)} y_i + aW(t)$$

where x_i denotes the income of insurance company; y_i stands the aggregate claims between times 0 and t . The other variables have the same definition as definition1.1.The model is called multiple line risk model perturbed by diffusion [3]

Definition1.4 Set

$$R(n) = u + cM(n) - S(n) \quad , \quad S(n) = \sum_{i=1}^{N(n)} Z_i, t \geq 0$$

where $\{M(n), n = 0,1,2,\Lambda\}$ is a Poisson stochastic variable with parameter λ ; $S(n)$ is compound binomial distribute. The model is called the ruin probabilities in the compound binomial risk model [4] .[3] discussed that Liver tumor division in restorative pictures has been generally considered as of late, of which the Level set models show an uncommon potential with the advantage of overall optima and functional effectiveness. The Gaussian mixture model (GMM) and Expected Maximization for liver tumor division are introduced.

The paper considers the diffusion at the base of [4]. Consider the factors not confirmed of insurance company .We get the ruin probability of the generalized compound binomial risk model by diffusion. It is a new model under the more general condition.

II. GENERALIZED COMPOUND BINOMIAL RISK MODEL

Definition 2.1 Suppose $u \geq 0, c > 0$, all of the following stochastic variables are on (Ω, F, P) of a completed probability space.

(1) $\{M(n), n = 0,1,2,\Lambda\}$ is a Poisson stochastic variable with parameter λ ;

(2)$\{Z_i, i = 1,2,\Lambda\}$ is the stochastic variable array of i. i. d between 0 and $+\infty$;

(3) $\{W(n), n = 0,1,2,\Lambda\}$ is a standard Brownian motion, it represents the income and payment not

confirmed of the insurance company, a is a positive constant;

(4) $\{N(n), n = 0,1,2,\Lambda\}$ is a binomial stochastic array with parameter p and $\{M(n), n = 0,1,2,\Lambda\}$, $\{N(n), n = 0,1,2,\Lambda\}$,{ Z_i , $i = 1,2,\Lambda$ } , $\{W(n), n = 0,1,2,\Lambda\}$ are independent each other.

Set

$$R(n) = u + cM(n) - S(n) + aW(n),$$

$$S(n) = \sum_{i=1}^{N(n)} Z_i, n = 0,1,2,\Lambda$$

$\{R(n)\}_{n=0}^{\infty}$ is called generalized compound binomial risk model by diffusion.

The real background of the model is: in the affairs of the insurance company, $u(u \ge 0)$ is the initial capital; c is the rate at which the premium is received. $M(n)$ shows the count of insurance at time interval $(0, n]$,it is a Poisson distribute with parameter $n\lambda$, it is the only income of insurance company. $W(n)$ is a standard Brownian motion which stands the income and payment not confirmed of a insurance company. $\{Z_i\}_{i=1}^{\infty}$ is the random variable array that is i. i. d at R^+. $N(n)$ is total number of times constantly till time n, $S(n)$ is aggregate claim till time n, it is the only expenditure of a company that the company settles a claim after the accident happened to the insured. $R(n)$ is the surplus capital of a insurance company at moment n.

Set

$$T = \inf\{n | R(n) < 0\}$$

denotes the clock of ruin ,so

$$\psi(u) = P(T < \infty)$$

denotes ruin probability which is the function of initial value u.

The paper discuss the characters of surplus process, and we get two theory of the ruin probability in this new model.

III. CHARACTERS OF SURPLUS PROCESS

Character 3.1 surplus process $\{R(n), n \ge 0\}$ is a stationary independent increments process.

Proof For $n_0 < n_1 < \Lambda < n_n$

$$R(n_i) - R(n_{i-1}) = c[M(n_i) - M(n_{i-1})]$$
$$-[S(n_i) - S(n_{i-1})] + a[W(n_i) - W(n_{i-1})]$$

$S(n_1) - S(n_0), S(n_2) - S(n_1), \Lambda$, $S(n_n) - S(n_{n-1})$, $W(n_1) - W(n_0), W(n_2) - W(n_1), \Lambda$, $W(n_n) - W(n_{n-1})$, $M(n_1) - M(n_0), M(n_2) - M(n_1), \Lambda$, $M(n_n) - M(n_{n-1})$ are independent each other ,so surplus process $\{R(n), n \ge 0\}$ is a independent increments process.

Because of

$$R(n+s) - R(n) = c[M(n+s) - M(n)]$$
$$-[S(n+s) - S(n)] + a[W(n+s) - W(n)]\cdot$$

For any $n \ge 0$, $M(n+s) - M(n)$, $S(n+s) - S(n)$, $W(n+s) - W(n)$ have the same distribution .And For any $n \ge 0$, $R(n+s) - R(n)$ have the same distribution too .So $\{R(n), n \ge 0\}$ is a stationary process.

So surplus process $\{R(n), n \ge 0\}$ is a stationary independent increments process.

Character 3.2 $E[R(n)] = u + (c\lambda - \mu p)n$,

$$Var[R(n)] = \lambda n + np(q\mu^2 + \sigma^2) + a^2 n.$$

Where $\mu = E(Z)$, $\sigma^2 = Var(Z)$.

Proof For any n, we have

$$E[R(n)] = E[u + cM(n) - S(t) + aW(t)]$$
$$= u + cE[M(n)] - E[S(n)] + aE[W(n)]$$

where

$E[W(n)] = 0$, $E[M(n)] = \lambda n$, $E[S(n)] = n\mu p$

so we get

$$E[R(n)] = u + (c\lambda - \mu p)n$$

$\{M(n), n \ge 0\}$, $\{S(n), n \ge 0\}$ and $\{W(n), n \ge 0\}$ are independent ,so we have

$$Var[R(n)] = c^2 Var[M(n)] + Var[S(n)] + a^2 Var[W(n)].$$

And

$$Var[W(n)] = n, \quad Var[M(n)] = \lambda n$$

$$Var[S(n)] = E\{Var[S(n)|N(n)]\} + Var\{E[S(n)|N(n)]\}$$
$$= np(q\mu^2 + \sigma^2)$$

so we have

$$Var[R(n)] = \lambda n + np(q\mu^2 + \sigma^2) + a^2 n$$

In order to guarantee the stability of the insurance company is managed, the income of the insurance premium should be greater than the sum of claim within unit time. According to nature 3.2 , need supposing $c\lambda > \mu p$.

IV. RUIN PROBABILITY

Definition4.1 Suppose $M_Z(r)$ is square mother's function of Z_i and

$$c\lambda(e^{-r} - 1) + \ln[pM_Z(r) + q] + \frac{1}{2}a^2 r^2 = 0 \qquad (1)$$

R is not zero positive solving which is called adjustment coefficient ,and equation (1) is called adjustment coefficient equation.

Lemma4.2 The solving of the adjustment coefficient equation (1) is only exit.

Proof Set

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$$g(r)=c\lambda(e^{-r}-1)+\ln[pM_Z(r)+q]+\frac{1}{2}a^2r^2$$

So, we have

$$\frac{dg(r)}{dr}=-c\lambda e^{-r}+\frac{pE[Ze^{rZ}]}{pM_Z(r)+q}+a^2r$$

$$\frac{d^2g(r)}{dr^2}=c\lambda e^{-r}+$$

$$\frac{pE[Z^2e^{rZ}][pM_z(r)+q]-p^2E^2[Ze^{rZ}]}{[pM_Z(r)+q]^2}+a^2$$

We have $g''(r)>0$ by Schwartz inequality, so $g(r)$ is downward projection function, equation (1) has two solving at most .It is obvious $g(0)=0$, so $r=0$ is zero solving. And because of

$$\frac{dg(r)}{dr}\Big|_{r=0}=-c\lambda+\mu p<0,$$

and as $r\to+\infty$, $g(r)\to+\infty$. So the positive solving R is only exit

The theorem following provides the general expression formula of ruin probability.

Theorem 4.3 $\psi(u)=\dfrac{e^{-Ru}}{E[e^{-R\cdot R(T)}|T<\infty]}$,

Where R is adjustment coefficient.

Proof For any $u\ge0$, $r>0$, we have

$$E[e^{-r\cdot R(n)}]=E[e^{-r\cdot R(n)}|T\le n]P(T\le n)$$
$$+E[e^{-rR(n)}|T>n]P(T>n) \qquad (2)$$

Because of

$$R(n)=u+cM(n)-S(n)+aW(n),$$

we have

$$E[e^{-r\cdot R(n)}]\}=\exp\{-ru+c\lambda n(e^{-r}-1)+$$
$$n\ln[pM_Z(r)+q]+\frac{1}{2}a^2r^2n\}$$

When R is the not zero positive solving of the equation (1), we get

$$E[e^{-R\cdot R(n)}]=e^{-Ru}.$$

When r is replaced by R in (2), we have

$$e^{-Ru}=E[e^{-RR(n)}|T\le n]P(T\le n)$$
$$+E[e^{-RR(n)}|T>n]P(T>n) \qquad (3)$$

Surplus is

$$R(n)=R(T)+[R(n)-R(T)]$$
$$=R(T)+c[M(n)-M(T)]]$$
$$-[S(n)-S(T)]+a[W(n)-W(T)]。$$

When $T<n$, we get that $M(n)-M(T)$, $S(n)-S(T)$, $W(n)-W(T)$ and $R(T)$ are independent each other by definition 2.1. $M(n)-M(T)$ is a Poisson distribution with parameter $\lambda(n-T)$, $S(n)-S(T)$ is a compound binomial distribution with parameter $n-T$ and p ,and $W(n)-W(T)$ is $N(0,n-T)$ .So

$$E[e^{-R\cdot R(n)}|T\le n]$$
$$=E\{e^{-R\cdot R(T)}\cdot\cdot e^{-Rc[M(n)-M(T)]}\cdot e^{R[S(n)-S(T)]}\cdot$$
$$e^{-Ra[W(n)-W(T)]}|T\le n\}$$
$$=\exp\{-RR(T)+c\lambda n(e^{-R}-1)$$
$$+n\ln[pM_Z(R)+q]+\frac{1}{2}a^2R^2n|T<n\}$$
$$=E[e^{-R\cdot R(T)}|T\le n].$$

And then, we have

$$\lim_{n\to\infty}E[e^{-R\cdot R(n)}|T\le n]P(T\le n)$$
$$=E[e^{-R\cdot R(T)}|T\le\infty]\,\psi(u).$$

To item 2 right of (3) by character 3.2 we have

$$E[R(n)]=u+(c\lambda-\mu p)n,$$
$$Var[R(n)]=\lambda n+np(q\mu^2+\sigma^2)+a^2n。$$

For any $n\ge0$, we consider

$$q(n)=u+(c\lambda-\mu p)n-\sqrt{\lambda+p(q\mu^2+\sigma^2)+a^2}\,n^{\frac{2}{3}}.$$

Because of $c\lambda>\mu p$, we have $q(n)>0$ when n is abundant and big. So we get

$$E[e^{-R\cdot R(n)}|T>n]=$$
$$E[e^{-R\cdot R(n)}|T>n,0<R(n)\le q(n)]P[0<R(n)\le q(n)]\quad+$$
$$E[e^{-R\cdot R(n)}|T>n,R(n)>q(n)]P[R(n)>q(n)]\le$$
$$P(0<R(n)\le q(n))+\exp\{-R\cdot q(n)\}$$

We get $\lim_{n\to\infty}\exp\{-R\cdot q(n)\}=0$. And apply Chebychev inequality, we have

$$P(0<R(n)\le q(n))=$$
$$P(0<R(n)\le E[R(n)-\sqrt{\lambda+p(q\mu^2+\sigma^2)+a^2}\,n^{\frac{2}{3}}]\le$$
$$P(|R(n)-E(R(n))|\ge\sqrt{\lambda+p(q\mu^2+\sigma^2)+a^2}\;n^{\frac{2}{3}})\le$$
$$\frac{Var[R(n)]}{[\lambda+p(q\mu^2+\sigma^2)+a^2]n^{4/3}}=n^{-1/3}\quad。$$

So we get

$$\lim_{n\to\infty}P(0<R(t)\le q(t))=0.$$

In sum , let $n\to\infty$ at both ends of (2) , we have

$$e^{-Ru}=E[e^{-R\cdot R(T)}|T<\infty]\psi(u),$$

That is to say

$$\psi(u)=\frac{e^{-Ru}}{E[e^{-R\cdot R(T)}|T<\infty]}.$$

Now the theorem following provides an upper

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bound of ruin probability.

Theorem4 .4 $\psi(u) < e^{-Ru}$.

Proof When $T < \infty$, $R(T) < 0$, so $E[e^{-R \cdot R(T)} | T < \infty] > 1$,then

$$\psi(u) < e^{-Ru}.$$

The inequality is called lundberg inequality. It is a very important inequality of ruin model. It is obvious that ruin probability is not easy get by theorem 4.3.

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