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# Integration Teaching for ICS in Multi-Disciplinary Background—a Nonlinear Case Study

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***Abstract*—For the mathematics course of Information and Computing Science (ICS) speciality, in the traditional teaching, it is often explained only from the theory and limited to the scope of theory. However, mathematical theory comes from real life. It is actually the result of multi-disciplinary and multi professional cooperation. This paper discusses the teaching method of mathematics course of ICS, and puts forward the method of multi-disciplinary integration in practical teaching. This paper gives an example by using chaotic nonlinear system to illustrate how to carry out multi-disciplinary teaching.**

***Index Terms*—ICS, integration teaching, multi-disciplinary, nonlinearity.**

## I. Introduction

In the traditional mathematics teaching, teachers often explain simply from the theory and think limited to the theoretical category. Mathematics should be the result of the cooperation and integration of multi-disciplinary and professional. In the classroom, if teachers only explain from the theory, because students lack the corresponding perceptual knowledge, and cannot go deep into the results and promotion of multi-disciplinary cooperation, this traditional teaching model cannot adapt to the development of science. Therefore, we propose a multi-disciplinary integrated teaching model. In order to realize multi-disciplinary integrated teaching, it is first necessary to establish a teacher team with multi-disciplinary background, so as to be able to carry out multi-disciplinary teaching, such as having several professional knowledge: Physics, Mathematics, Computer, Cybernetics, etc. Specifically, understand the principle of physical model, be able to use mathematical knowledge for derivation and operation, and solve relevant problems in combination with control theory and experiment with computer programming. In this way, a multi-disciplinary teaching team will be formed. Multi-disciplinary personnel should be comprehensive and diverse enough and not be limited. College education needs to cultivate professional talents. At the same time, we should strengthen the cultivation of compound talents with a variety of professional knowledge, and expand students' knowledge and ability in depth and breadth, as well as the cultivation of our ICS students [1]-[4].

Therefore, based on the author's teaching experience, this paper puts forward the method of multi-disciplinary integration for the course of ICS, takes a nonlinear problem as an example, and shows how to solve the problems of multi-disciplinary integration in teaching.

## II. Physical-Mathematical Modelling

In physical models, there are many chaotic models, e.g., Lorenz, Rössler, Chua's circuit [5]-[7]. Here, we consider a chaotic model and will use control theory to solve the chaotic synchronization problem. The drive model is given by

$$\dot{x}(t) = Ax(t) + f(x), \quad x(0) = x_0 \tag{1}$$

and the response model given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t-\tau) + f(\hat{x}), \quad \hat{x}(0) = \hat{x}_0 \tag{2}$$

where $f: R^n \to R^n$ is a continuous nonlinear function satisfying Lipschitz condition, $x, \hat{x} \in R^n$ are the state vectors, $u \in R$ is the input, $\tau > 0$ is a constant time-delay from the controller to the actuator. The matrices $A \in R^{n\times n}$ and $B \in R^{n\times 1}$ are constant and the pair $(A, B)$ is controllable.

## III. Synchronization Using Control Theory

Let the synchronization error be $e(t) = \hat{x}(t) - x(t)$. Differentiating both sides of the error equality and substituting the models (1) and (2) into the result, respectively, yield the error model

$$\dot{e}(t) = Ae(t) + B_0 u(t-\tau) + f(\hat{x}) - f(x), \quad e(0) = e_0 \tag{3}$$

with $u(t) \equiv 0$ when $t \in [-\tau, 0)$. Define $B = \mathrm{e}^{-\tau A} B_0$ and a new variable

$$z(t) = e(t) + \int_{t-\tau}^{t} \mathrm{e}^{A(t-s)} Bu(s) \mathrm{d}s \tag{4}$$

It follows that $z(0) = e(0)$. Then, the time-delay model (3) is equivalent to the following delay-free one [8]

$$\dot{z}(t) = Az(t) + Bu(t) + f(\hat{x}) - f(x), \quad z(0) = z_0 \tag{5}$$

Our purpose is to design a controller guaranteeing synchronization error (5) asymptotically stable.

**Theorem 1.** Consider the chaotic drive model (1) and the response model (2). Given the nonlinear control

$$u(t) = -R^{-1}B^{\mathrm{T}}Pz(t) - k\|z(t)\| \operatorname{sign}\left(R^{-1}B^{\mathrm{T}}Pz(t)\right) \tag{6}$$

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where $\|\cdot\|$ denotes the Euclidean norm, $P$ is positive definite and unique solution of Riccati equation

$$A^{\mathrm{T}}P+PA-2PBR^{-1}B^{\mathrm{T}}P+Q=0 \tag{7}$$

$Q$ is a positive semi-definite matrix, and the positive real number $k$ satisfies $k\geq L\big/\sqrt{\lambda_{\min}(PBR^{-1}B^{\mathrm{T}})}$, in which $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalues of a matrix, then, the synchronization error is asymptotically stable.

**Proof**. Substituting the controller (6) into the delay-free error model (5) yields the closed-loop model

$$\begin{aligned}\dot{z}(t)&=(A-BR^{-1}B^{\mathrm{T}}P)z(t)+f(\hat{x})-f(x)\\&\quad-k\|z(t)\|B\operatorname{sign}\left(R^{-1}B^{\mathrm{T}}Pz(t)\right)\end{aligned} \tag{8}$$

Choosing a Lyapunov candidate function $V=z^{\mathrm{T}}(t)Pz(t)$, the derivative of it along the model (5) is

$$\begin{aligned}\dot{V}&=\dot{z}^{\mathrm{T}}(t)Pz(t)+z^{\mathrm{T}}(t)P\dot{z}(t)\\&=z^{\mathrm{T}}(t)\left(A^{\mathrm{T}}P+PA-2PBR^{-1}B^{\mathrm{T}}P\right)z(t)\\&\quad+2z^{\mathrm{T}}(t)P\left[f(\hat{x})-f(x)\right]\\&\quad-2kPBR^{-1}B^{\mathrm{T}}P\|z(t)\|z^{\mathrm{T}}(t)\operatorname{sign}\left(R^{-1}B^{\mathrm{T}}Pz(t)\right)\end{aligned} \tag{9}$$

Due to (7), (9) deduces

$$\dot{V}\leq-z^{\mathrm{T}}(t)Qz(t)-2\left[k\left\|PBR^{-1}B^{\mathrm{T}}\right\|-L\right]\|P\|\|z(t)\|^{2} \tag{10}$$

Taking $k\geq L\big/\sqrt{\lambda_{\min}(PBR^{-1}B^{\mathrm{T}})}$, (10) is negative, which implies that the closed-loop model (8) is asymptotically stable; in other terms, as $t\to\infty$, $z(t)\to 0$. For (4), as $t\to\infty$, $e(t)\to 0$. The proof of Theorem 1 is completed. □

Specifically, we consider the chaotic Rössler model described by

$$\begin{aligned}\dot{x}_1(t)&=-x_2(t)-x_3(t)\\\dot{x}_2(t)&=x_1(t)+ax_2(t)\\\dot{x}_3(t)&=-cx_3(t)+x_1(t)x_3(t)+b\end{aligned} \tag{11}$$

which has a chaotic attractor when $a=0.2, b=0.2, c=10$. The controlled synchronization model is

$$\begin{aligned}\dot{\hat{x}}_1(t)&=-\hat{x}_2(t)-\hat{x}_3(t)+u(t-\tau)\\\dot{\hat{x}}_2(t)&=\hat{x}_1(t)+a\hat{x}_2(t)+u(t-\tau)\\\dot{\hat{x}}_3(t)&=-c\hat{x}_3(t)+\hat{x}_1(t)\hat{x}_3(t)+b+u(t-\tau)\end{aligned} \tag{12}$$

Subtracting (12) from (11) yields the synchronization error model

$$\begin{aligned}\dot{e}_1(t)&=-e_2(t)-e_3(t)+u(t-\tau)\\\dot{e}_2(t)&=e_1(t)+ae_2(t)+u(t-\tau)\\\dot{e}_3(t)&=-ce_3(t)+\hat{x}_1(t)\hat{x}_3(t)-x_1(t)x_3(t)+u(t-\tau)\end{aligned} \tag{13}$$

which has the constant time-delay $\tau$ and $B_0=\begin{bmatrix}1 & 1 & 1\end{bmatrix}^{\mathrm{T}}$. Through the functional transformation (4), the time-delay model (13) is converted into the equivalent delay-free one

$$\begin{aligned}\dot{z}_1(t)&=-z_2(t)-z_3(t)+b_1u(t)\\\dot{z}_2(t)&=z_1(t)+az_2(t)+b_2u(t)\\\dot{z}_3(t)&=-cz_3(t)+\hat{x}_1(t)\hat{x}_3(t)-x_1(t)x_3(t)+b_3u(t)\end{aligned} \tag{14}$$

where $B\triangleq\begin{bmatrix}b_1 & b_2 & b_3\end{bmatrix}^{\mathrm{T}}=e^{-\tau A}B_0$. Take $R=1/2$, $Q=I$, $L=1$, and $k=0.5774$. Then, the solution $P$ of Riccati equation (7) can be solved, so does the controller (6). According to Theorem 1, the exponential stability of the synchronization error can be guaranteed.

## IV. Simulation Using Computer Programming

By using computer programming, we will verify the effectiveness of the designed controllers in simulation.

Take the parameter values as $a=0.2, b=0.2, c=10$ in Rössler system (11). The Rössler chaos are illustrated in Fig. 1.

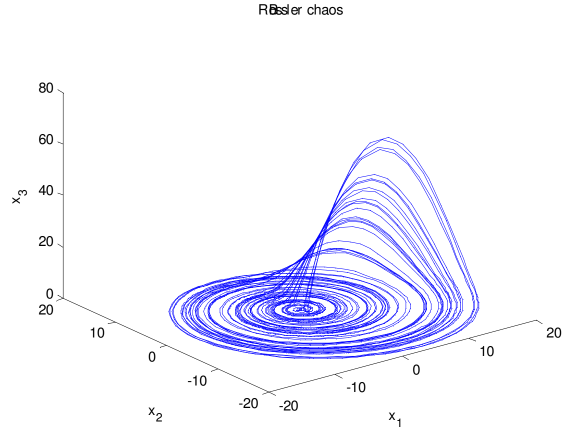


Fig. 1. The chaotic Rössler .

Set the time delay $\tau=0.01\mathrm{s}$ for the synchronization mode (12). The synchronization error model (13) and the equivalent delay-free one (14) can be achieved, as well as the controller (6). The matrices values are calculated as follows:

$$B=\begin{bmatrix}1.0205\\0.9879\\1.1052\end{bmatrix},\quad P=\begin{bmatrix}0.9764 & -0.3512 & -0.0940\\-0.3512 & 1.9030 & 0.0434\\-0.0940 & 0.0434 & 0.0594\end{bmatrix}$$

$$R^{-1}B^{\mathrm{T}}P=\begin{bmatrix}0.5456 & 1.5696 & 0.0126\end{bmatrix}$$

Set the initial states as

$$\begin{aligned}&\left(x_1(0),x_2(0),x_3(0),\hat{x}_1(0),\hat{x}_2(0),\hat{x}_3(0),z_1(0),z_2(0),z_3(0)\right)\\&=(0,0.1,0.1,0,0.2,0.2,0,0.1,0.1)\end{aligned}$$

The simulation is performed by using Matlab programming. The synchronized states of chaotic Rössler are shown in Fig. 2. The synchronization errors are shown in Fig. 3, where it can be observed that at about 6sec all the synchronization errors converged to zeroes.

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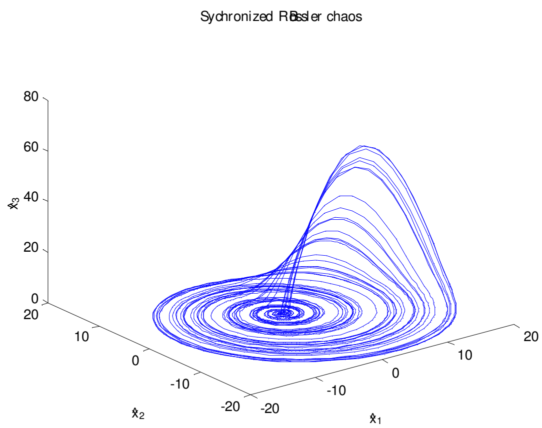

Fig. 2. Synchronized chaotic Rössler .

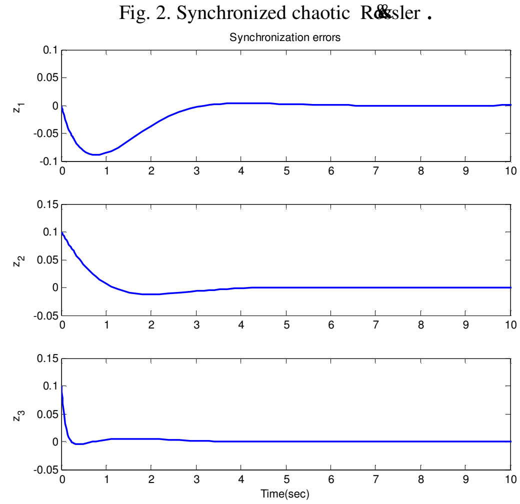

Fig. 3. Synchronization errors.

## V. Conclusion

By using the knowledge of physics, mathematics, computer and cybernetics, this paper explains how to solve the problem of synchronous control of chaotic nonlinear model, and shows the method of multi-disciplinary integration teaching and its benefits. If traditional teaching methods are used, similar problems cannot be shown how to solve to students, and students' cognition cannot be fully expanded. Teachers with multi-disciplinary professional knowledge providing the basis for the realization of multi-disciplinary integrated teaching methods, can carry out multi-disciplinary integrated teaching of ICS speciality, solve students' difficult problems, and improve the teaching quality and students' professional ability as a whole.

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