



# Discussion on the Teaching Mode of Linear Algebra Based on Application Examples

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**Abstract**—In this paper, aiming at the phenomenon that linear algebra generally emphasizes theory, neglects practical application, and separates concepts, definitions, principles and practical applications, a teaching mode of linear algebra based on application examples is put forward, that is, the teaching of linear algebra should adopt the teaching mode of integrating theory with practice, guide students to learn basic theories, and apply what they have learned to solve practical problems, so as to improve students' thirst for knowledge and better complete teaching tasks.

**Index Terms**—Examples, Linear algebra, model of instruction.

## I. INTRODUCTION

Linear algebra is an important basic course of mathematics offered by applied disciplines such as economics, management, science and engineering, etc., which plays a great role in cultivating college students' thinking ability, calculation ability, induction ability and analogical reasoning ability. As a basic course, linear algebra has nothing more than two contents in the teaching system of higher education. First, learn basic knowledge and theory, and understand the knowledge content of linear algebra itself; The second is to solve practical problems [1]. Therefore, how to use the basic knowledge of linear algebra that we have learned to solve practical problems in real life is a problem that we have been thinking about in the teaching process.

For students majoring in science and engineering, linear algebra is an indispensable basic course of mathematics, which plays a vital role in students' future development. In the actual linear algebra teaching, the related teaching activities to improve students' mathematical ability cover two aspects. First, the teaching of basic knowledge, that is, teaching a lot of basic knowledge of mathematics; Secondly, it is to take the acquired mathematical knowledge to have a good solution to practical problems, that is, to cultivate the practical ability to solve practical problems [2]. Facing the current era background of knowledge explosion, quality education also pays more attention to students' application ability.

In the teaching of linear algebra in most universities of

science and engineering, there are still many drawbacks

that need to be rectified, which can be summarized in the following two aspects: First, because students have just entered the university, they have not been able to break away from the previous exam-oriented education environment. Therefore, in most cases, during the actual teaching period, they can realize that they do not have good independent thinking ability, and in the course of learning, they can't really grasp the meaning of learning related knowledge points, so they often just apply formulas mechanically in the process of solving problems, lacking profound cognitive understanding, and they can't use them flexibly on the basis of understanding. In this case, it is difficult for students to systematically learn about relevant knowledge points during the study of linear algebra, let alone to apply what they have learned to solve problems in practice. Secondly, linear algebra is a course with mutual penetration, crisscrossing and close relationship between knowledge before and after. This course has a wide range of applications, and it also has more flexible ways to deal with specific problems. At present, most of the applied teaching materials only pay attention to the study of theory, but the practical application does not pay much attention to it. Therefore, the phenomenon of "attaching importance to formula derivation and neglecting numerical calculation" appears, which can't be adapted to the purpose of application in engineering teaching, and will be used as an important principle of measuring degree, and rarely can be closely combined with other subject knowledge and real-life examples [3]. In addition, in the existing teaching mode, it is easy to ignore the application background of principles and examples.

## II. TEACHING MODE OF LINEAR ALGEBRA BASED ON APPLICATION EXAMPLES

### A. Teach concepts through examples

In the teaching process of linear algebra, we must compare, associate and abstract to get the definition [4]. For example, in



the after-class questions of most linear algebra textbooks, Kramer's rule is required to solve linear equations:

$$\begin{cases} x_1 - 2x_2 + x_3 + x_4 = -1 \\ -x_2 + 3x_3 - x_4 = 0 \\ x_1 + x_2 - x_3 + x_4 = -2 \\ x_1 + 3x_3 - x_4 = 1. \end{cases}$$

To find a solution, we need to calculate the value of the coefficient determinant first. In textbooks, we usually calculate the second-order and third-order determinants by diagonal rule, and then give the definition of the n-order determinant. In teaching, if we don't emphasize the essential characteristics of determinants and the application scope of diagonal rule, it will easily lead to students' illusion that all determinants can be calculated by diagonal rule.

If the fourth-order determinant is calculated by diagonal rule, the solution of the equations will be wrong, because the essence of determinant definition is not appreciated [5]. If we start with the second-order and third-order determinants in the classroom teaching of n-order determinant definition, we can guide students to think about the characteristics of determinants: the number of determinants; There are  $2!$  items in the second-order determinant expression, the third-order determinant expression has  $3!$  items, n-order determinant expression has  $n!$  items; Each term consists of the product of n elements from different rows and columns, with the number of positive and negative terms accounting for half. Besides, it must be emphasized that "finding determinant by diagonal rule" is not a general method of determinant calculation, but it is special, and it is not suitable for the calculation of determinant above the third order. In classroom teaching, students can first ask why diagonal method is not suitable for the calculation of determinant above the third order, so as to deepen their impression.

#### B. Select examples with professional background

The requirement and purpose of linear algebra teaching is to gradually cultivate students' abstract thinking ability, logical reasoning ability, spatial imagination ability and self-study ability through various teaching links while imparting knowledge, and pay special attention to cultivating students' skilled operation ability and the ability to analyze and solve problems by comprehensively applying the knowledge they have learned. Linear algebra teaching for engineering students, Simply explaining the subtleties of theorems without connecting with practice or enumerating vivid examples is difficult to produce interactive teaching scenes. Most of the teachers who teach linear algebra graduated from mathematics major and have profound professional basic theoretical knowledge. However, due to the limitation of engineering background, knowledge and class hours, most of them only teach knowledge points in textbooks. This teaching method can not effectively guide students to learn actively, and can not achieve good teaching effect. Especially in today's quality education, in order to reverse this embarrassing situation, teachers need to contact

life more widely. In the teaching process, it is necessary to organize the course teaching from problems rather than definitions directly. First, ask some questions that can arouse students' interest, and then gradually lead out concepts and methods, so that students can master knowledge points and know how to use knowledge points to solve practical problems [6]. When teaching this course for different majors, we should give more examples related to their majors. For example, we can explain more examples of production costs and management decisions for management majors; Explain more examples of the application of information coding and linear algebra in programming for students majoring in information engineering[6]. Only in this way can students' ability to turn practical problems into mathematical models be improved, their learning space be expanded, and their practical ability and innovative spirit be improved. How to realize the above expectations requires teachers to make daily teaching and academic exchanges.

Only rich teaching materials can properly set up mathematical scenarios, put forward mathematical problems from different angles, arouse students' enthusiasm for learning linear algebra and improve the active atmosphere in the classroom. For example, when talking about the concept of matrix, first tell students that the definition of matrix is the number table, and explain its similarities and differences with determinants. In order to further broaden students' vision and enthusiasm for learning linear algebra, we can put forward "

For example, the gray image is a matrix and each element is an integer from 0 to 255; The computer screen is also an example closely related to the matrix. Then, it is introduced that the color of the screen is the compound of three primary colors of red, green and blue, that is, the compound of three  $1024 \times 768$  matrices (commonly used computer screens). These examples can make students realize the ubiquitous matrices in real life, broaden students' knowledge and improve students' interest in learning. Then, we return to the question of why matrices are introduced into mathematics-in order to express equations and solve equations. Using Grammer Rules to discuss the solution of equations, the linear equations are characterized by the fact that the number of equations and unknowns must be the same, because the determinant used in them is limited. "Can we still use Grammer Rules to discuss the solution of equations with unknowns and equations?" This question makes students aware of the application scope of Grammer Rules, and also lays a good foundation for learning how to apply the matrix to solve equations. This will form a bridge from the practical application background to the correlation of mathematical theory, and also make students understand that the matrix is useful.

#### C. Combined with the teaching example of mathematical modeling

Introducing the idea of mathematical modeling into the example teaching of linear algebra will increase the interest and taste of knowledge, improve students' learning motivation, and better help students grasp the content of teaching and learning. For example, in the teaching of



moment matrix, we can add cryptography and predict the development trend of things, construct a model based on examples, and introduce relevant teaching contents. In the online teaching of linear equations, we can construct the model and introduce the concept explanation by plugging in the network flow questions and applying examples of economic and economic balance. The general steps are: question raising → model building → concept introduction → model solving.

The following is a teaching example of "information encryption problem" combined with mathematical modeling.

The first step, the question is put forward: in order to prevent information from being leaked during transmission, it is usually necessary to encrypt it. To transmit the information "very good" now, how do you encrypt and transmit the information?

The second step is to establish the pattern: "very good" can be coded as 22 5 18 25 2 7 15 15 4, in which 2 represents a space, and other letters are represented by their sequential numbers in the alphabet. However, this encryption can be easily cracked, and the information can be further disguised by matrix multiplication. First, put the upper coding into a matrix, and write the matrix as

$$A_1 = \begin{pmatrix} 22 & 5 & 18 \\ 25 & 2 & 7 \\ 15 & 15 & 4 \end{pmatrix}.$$

The dense moment matrix is  $A_2 = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ -1 & -1 & 1 \end{pmatrix}$ .

Their flight is

$$A_1 A_2 = \begin{pmatrix} 22 & 5 & 18 \\ 25 & 2 & 7 \\ 15 & 15 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ -1 & -1 & 1 \end{pmatrix} \\ = \begin{pmatrix} 41 & 14 & 45 \\ 49 & 22 & 34 \\ 71 & 41 & 34 \end{pmatrix}.$$

$A_1 A_2$  times a matrix  $A$  to the right, make  
So the encrypted code is 41 14 45 49 22 34 71 41 34.  
 $A_1 A_2 A = A_1$ .

Find a matrix  $A$ , make  $A_2 A = E$ . It can be decrypted.

The third step, concept introduction: the definition of inverse matrix.

The fourth Step, model solving: in this example, the matrix  $A$  to be found is the inverse matrix of the square matrix  $A_2$ . And all the elements in it are integers. When constructing the encryption matrix, identity matrix can be used to perform

the third elementary transformation several times, so that  $|A_2| = 1$ .

### III. CONCLUSION

In short, when explaining the definitions and theorems of linear algebra, we should pay attention to the ways of abstracting, comparing and setting the problem background. At the same time, we should be able to contact with application examples, so that students can gradually feel that linear algebra is no longer boring, and then have a strong thirst for knowledge.

Teachers should broaden their knowledge, strengthen exchanges, deepen accumulation and explore feasible and efficient classroom teaching methods of integrating theory with practice.

Facing the new trend that higher education is developing vertically and deeply, and the society is eager for high-quality practical talents, the linear algebra curriculum urgently needs to speed up the teaching reform, intensify the reform and implement the curriculum [7].

We need to actively learn from the experience of other curriculum reforms, break through the shackles of rigid system and rigid thinking with greater courage and broader vision, take the transformation of teaching mode as the starting point, and aim at timely adjustment of teaching content, timely updating of teaching methods and methods, flexible and diverse teaching means, promotion of hierarchical teaching and establishment of scientific evaluation system. The course of linear algebra will be truly built into a professional basic course with outstanding application, full of the needs of teaching, education and training, which is in line with students' learning characteristics and meets the requirements of the times.

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