

A STUDY ON PROPERTIES OF FUZZY ROUGH LOCALLY CLOSED SETS

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Abstract— The two important and mutually orthogonal features of imperfect data and knowledge are fuzzy sets and rough sets, where the fuzzy allows that object belong to a set or relation to a given degree and the Rough provides approximations of concepts with available incomplete information. The concepts of fuzzy rough semi open sets, fuzzy rough pre-open sets, fuzzy-rough α -open sets and fuzzy-rough β -open sets are introduced here. Along with it several interesting properties and interrelations are discussed with suitable examples. The concept on some properties of fuzzy rough locally closed set, fuzzy rough semi locally closed, fuzzy rough pre locally closed, fuzzy rough α -locally closed set, fuzzy rough β -locally closed set and fuzzy rough regular locally closed set are also introduced.

Index Terms— Fuzzy set theory, Rough set theory, fuzzy rough topology.

I. INTRODUCTION

Fuzzy set theory was first introduced by Zadeh (1965) [10]. It is the evolution of classic crisp logic into a multivariate, updated version. There are various advantages of Fuzzy over the crisp, the most important of which is that they have more flexible decision boundaries, allowing them to adapt to a certain area of application and more precisely reflect its particularities. In 1968, Chang introduced The idea of fuzzy topological space was developed to encompass many basic topological concepts such as open set, closed set, continuity, and compactness[2]. Fuzzy sets have numerous applications in the trending technologies like information theory [8] and control [9]. In Topological spaces, Bourbaki[1] introduced the concept of locally closed sets.

Rough set theory was proposed by Pawlak [6], Pawlak's rough set theory can be thought of as an analysis of concepts or facts through the use of knowledge bases. Obviously, the rough set theory is most useful in classification and inductive learning. Pawlak, on the other hand, discovered that an equivalence relation creates a partition and thus a topology (called Pawlak topology). It's also a brand-new data-reasoning mathematical method. It has been successfully applied to machine learning, intelligent systems, inductive reasoning, pattern recognition, meteorology, image processing, signal analysis, information discovery, decision analysis, expert systems, and a variety of real-life fields such as economics, medical diagnosis, biochemistry, environmental science, biology, chemistry, psychology, conflict analysis, medicine, pharmacology,

banking, market research, engineering, speech recognition, material science [18], information analysis [20], data analysis, data mining [16, 17], linguistics, networking and so on[7, 3, 5].

Rough set theory is a common and effective machine learning technique. Rough sets models, which have only recently been proposed, are created by combining various fuzzy generalizations. To handle data with continuous attributes and identify anomalies, fuzzy-rough set theory can now be used in combination with rough set theory. The fuzzy-rough set model has proven to be very useful in many application areas because it is one of the most powerful tool for analyzing inaccurate and ambiguous data. We discovered that there are still many challenging issues related to the different implementation areas of fuzzy-rough set theory based on this thorough analysis, which may lead to potential research studies.

II. PRELIMINARIES:

Few basic concepts are summarized below for guidance and as pre-preparation before moving on to the comprehensive analysis.

Definition 2.1[1]

A topological space is a mathematical space in which limits, continuity, and connectedness can be described.

A Topology on a set X is a set of subsets of X say τ with the following properties:

- \emptyset and $X \in \tau$.
- The union of the items of any sub collection of τ is in τ .
- The intersection of the items of any finite sub collection of τ is in τ .

Then there is the ordered pair (X, τ) is called topological space. Every element in τ is an open set in X . The complement of open set is closed set in X .

Definition 2.2[4]

Let (V, B) be a rough universe, with V being a non empty set and B being a Boolean sub algebra of a Boolean algebra of all V 's subsets. Let's call a rough set as $X = (X_L, X_U) \in B^2$ with $X_L \subseteq X_U$.

A fuzzy rough set in the set X is an object of the form $A = (A_L, A_U)$, where A_L and A_U be a pair of maps $A_L: X_L \rightarrow L$ and $A_U: X_U \rightarrow L$ with $A_L(x) \leq A_U(x)$, $\forall x \in X_L$ where (L, \leq) a fuzzy rough set in the set X is an object of the form $A = (A_L,$

A_U), where A_L and A_U are a pair of maps $A_L: X_L \rightarrow L$ and $A_U: X_U \rightarrow L$ with $A_L(x) \leq A_U(x)$, X_L , say a completed and totally distributive lattice with the least and greatest elements as 0 and 1, respectively with an involute order reversing operation:: $L \rightarrow L$.

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A fuzzy rough set in the set X is an object of the form $A = (A_L, A_U)$, where A_L and A_U be a pair of maps $A_L: X_L \rightarrow L$ and $A_U: X_U \rightarrow L$ with $A_L(x) \leq A_U(x)$, $\forall x \in X_L$ where (L, \leq) a fuzzy rough set in the set X is an object of the form $A = (A_L, A_U)$, where A_L and A_U are a pair of maps $A_L: X_L \rightarrow L$ and $A_U: X_U \rightarrow L$ with $A_L(x) \leq A_U(x)$, A_L where (L, \leq) is a fuzzy lattice, say a completed and totally distributive lattice with the least and greatest elements as 0 and 1, respectively with an involutive order reversing operation:: $L \rightarrow L$.

Definition 2.3[4]

For any two fuzzy rough sets in X say $A = (A_L, A_U)$ and $B = (B_L, B_U)$.

- $A \subseteq B$ if and only if $A_L(x) \leq B_L(x)$, $\forall x \in X_L$ and $A_U(x) \leq B_U(x)$, $\forall x \in X_U$.
- $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
- If $\{A_i: i \in J\}$ be any family of fuzzy rough sets in X , where $A_i = (A_{iL}, A_{iU})$ then
 - $E = \bigcup_i A_i$ where $E_L(x) = \bigvee A_{iL}(x)$, $\forall x \in X_L$ and $E_U(x) = \bigvee A_{iU}(x)$, $\forall x \in X_U$
 - $E = \bigcap_i A_i$ where $F_L(x) = \bigvee A_{iL}(x)$, $\forall x \in X_L$ and $F_U(x) = \bigvee A_{iU}(x)$, $\forall x \in X_U$.

Definition 2.4[4]

If A and B are fuzzy sets in X_L and X_U respectively where $X_L \subset X_U$. Then the restriction of B on X_L and the extension of A on X_U (denoted by $B_{>L}$ and $A_{<U}$) are defined by complement of a fuzzy rough set $A = (A_L, A_U)$ in X are denoted by $\bar{A} = ((\bar{A})_L, (\bar{A})_U)$ and defined by $(\bar{A})_L(x) = (A_{U>L})'(x)$, $\forall x \in X_L$ and $(\bar{A})_U(x) = (A_{L<U})'(x)$, $\forall x \in X_U$. For simplicity we write $(\bar{A})_L, (\bar{A})_U$ instead of $((\bar{A})_L, (\bar{A})_U)$.

For proceeding we use the following abbreviation for easy understanding.

- FR-OS- Fuzzy rough open set
- FR-CS- Fuzzy rough closed set.
- FR-SOS- Fuzzy rough semi open set
- FR-SCS fuzzy rough semi closed set
- FR-POS- Fuzzy rough pre open set
- FR-PCS fuzzy rough pre closed set
- FR- α -OS- Fuzzy rough α -open set
- FR- α -CS fuzzy rough α -closed set
- FR- β -OS- Fuzzy rough β -open set

- FR- β -CS fuzzy rough β -closed set
- FR-ROS- Fuzzy rough regular open set
- FR-RCS fuzzy rough regular closed set
- FRRLCS- Fuzzy rough regular locally closed set.
- FRLCS- Fuzzy rough locally closed set.
- FRSLCS- Fuzzy rough semi locally closed set
- FRPLCS- Fuzzy rough pre locally closed set.
- FR α LCS- Fuzzy rough α -locally closed set.
- FR β LCS- Fuzzy rough β -locally closed set.

Definition 2.5[4]

A fuzzy rough topology on a non-rough set $X = \{X_L, X_U\}$ is a family τ of the fuzzy rough set in X satisfying the following axioms.

- $\bar{0}, \bar{1} \in \tau$
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- $\bigcup G_i \in \tau$ for arbitrary family $\{G_i / i \in I\} \subseteq \tau$.

Then the ordered pair (X, τ) is the fuzzy rough topological space. Every member in τ is called FR-OS in X . The complement of a FR-OS is called a FR-CS in X .

Definition 2.6[4]

Let A be fuzzy rough set in fuzzy rough topological space (X, τ) . Then,

- $\text{int}(A) = \bigcup \{G / G \text{ is an FR-OS in } X \text{ and } G \subseteq A\}$ is called a fuzzy rough interior of A .
- $\text{cl}(A) = \bigcap \{G / G \text{ is an FR-CS in } X \text{ and } G \supseteq A\}$ is called a fuzzy rough closure of A .

1. PROPERTIES OF FUZZY ROUGH LOCALLY CLOSED SETS

Definition 3.1

A fuzzy rough set A of a fuzzy rough topological space (X, τ) is called

- FR-SOS if $A \subseteq \text{cl}(\text{int}(A))$,
- FR-POS if $A \subseteq \text{int}(\text{cl}(A))$,
- FR- α -OS if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$,
- FR- β -OS if $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$,
- FR-ROS if $A = \text{int}(\text{cl}(A))$.

Note 3.1: The complement of FR-SOS (respectively FR-POS, FR- α -OS, FR- β -OS and FR-ROS) is a FR-SCS (respectively FR-PCS, FR- α -CS, FR- β -CS and FR-RCS).

Proposition 3.1

Let (X, τ) be fuzzy rough topological space.

- If A is FR-OS then, A is FR-SOS.
- If A is FR-OS then, A is FR-POS.
- If A is FR-OS then, A is FR- α -CS.
- If A is FR-OS then, A is FR- β -OS.
- If A is FR-ROS then A is FR-OS.
- If A is fuzzy rough α -open set then, A is FR-SOS.
- If A is FR-POS then, A is fuzzy rough β -open set.

- viii) If A is both fuzzy rough open and closed sets if and only if A is fuzzy rough regular open and fuzzy rough regular closed set.

Proof: The proof is obvious.

Definition 3.2

Let (X, T) be fuzzy rough topological space. A fuzzy rough set A in X is called a

- FRLCS if $A = B \cap C$ where B is FR-OS and C is FR-CS in (X, T) .
- FRSLCS if $A = B \cap C$ where B is FR-SOS and C is FR-SCS in (X, T) .
- FRPLCS if $A = B \cap C$ where B is FR-POS and C is FR-PCS in (X, T) .
- FRRLCS if $A = B \cap C$ where B is FR-ROS and C is FR-RCS in (X, T) .
- $FR\alpha$ LCS if $A = B \cap C$ where B is FR- α -OS and C is FR- α -CS in (X, T) .
- $FR\beta$ LCS if $A = B \cap C$ where B is FR- β -OS and C is FR- β -CS in (X, T) .

Proposition 3.2

Every FRLCS is a FRSLCS.

Proof: Let A be FRLCS. That is, $A = B \cap C$, where B is FR-OS and C is FR-CS. By (i) of Proposition 3.1, A is FRSLCS.

Proposition 3.3

FRLCS is a $FR\alpha$ LCS.

Proof: Let A be FRLCS. That is, $A = B \cap C$ where B is FR-OS and C is FR-CS. By (iii) of Proposition 3.1, A is $FR\alpha$ LCS.

Remark: Converse of the Proposition 3.2 and Proposition 3.3 need not be true shown in the following example.

Example 3.1

Let $X = \{X_L, X_U\}$ be rough set, where $X_L = \{a, b\}$ and $X_U = \{a, b, c\}$. Let $A = \{(a/.3, b/.4), (a/.4, b/.5, c/.6)\}$, $B = \{(a/.3, b/.5), (a/.4, b/.5, c/.7)\}$, $E = \{(a/.3, b/.45), (a/.4, b/.5, c/.6)\}$ and $F = \{(a/.6, b/.5), (a/.7, b/.55, c/.1)\}$ be any four FR-OS in X . The family $T = \{\bar{0}, \bar{1}, A, B\}$ is a Fuzzy rough topology on X . Then, the ordered pair (X, T) is fuzzy rough topological space. Now, $C = E \cap F$ is FRSLCS (respectively $FR\alpha$ LCS) but C is not FRLCS.

Proposition 3.4

Every FRLCS is a FRPLCS.

Proof: Let A be FRLCS. That is, $A = B \cap C$, where B is FR-OS and C is FR-CS. By (ii) of Proposition 3.1, A is FRPLCS in (X, T) .

Proposition 3.5

Every FRLCS is a $FR\beta$ LCS.

Proof: Let A be FRLCS. That is, $A = B \cap C$ where B is FR-OS and C is FR-CS. By (iv) of Proposition 3.1, A is $FR\beta$ LCS.

Remark: Converse of the Proposition 3.4 and Proposition 3.5 need not be true shown in the following example.

Example 3.2

Let $X = \{X_L, X_U\}$ be rough set, where $X_L = \{a, b\}$ and $X_U = \{a, b, c\}$.

Let $A = \{(a/.3, b/.4), (a/.4, b/.5, c/.6)\}$, $B = \{(a/.3, b/.5), (a/.4, b/.5, c/.7)\}$, $D = \{(a/.3, b/.5), (a/.4, b/.5, c/.6)\}$ and $F = \{(a/.4, b/.5), (a/.4, b/.5, c/.1)\}$ be any four fuzzy rough open sets in X .

The family $T = \{\bar{0}, \bar{1}, A, B\}$ is a Fuzzy rough topology on X . Then, the ordered pair (X, T) is fuzzy rough topological space. Now, $C = D \cap F$ is FRPLCS (respectively $FR\beta$ LCS) but C is not FRLCS.

Proposition 3.6

Every FRRLCS is a FRLCS.

Proof:

Let A be any FRRLCS. That is, $A = B \cap C$ where B is FR-ROS and C is FR-RCS. By (viii) of Proposition 3.1, A is FRLCS in (X, T) .

Proposition 3.7

Every FRRLCS is a FRPLCS.

Proof: Let A be Fuzzy FRRLCS. That is, $A = B \cap C$ where B is FR-ROS and C is FR-RCS. By (viii) of Proposition 3.1, A is FRLCS. Then by Proposition 3.4, A is FRPLCS.

Proposition 3.8

Every FRRLCS is a FRSLCS.

Proof: Let A be FRRLCS. That is, $A = B \cap C$ where B is FR-ROS and C is FR-RCS. By (viii) of Proposition 3.1, A is FRLCS. By Proposition 3.2, A is FRSLCS.

Proposition 3.9

Every FRRLCS is a $FR\alpha$ LCS.

Proof: Let A be FRRLCS. That is, $A = B \cap C$ where B is FR-ROS and C is FR-RCS. By (viii) of Proposition 3.1, A is FRLCS. By Proposition 3.3, A is $FR\alpha$ LCS.

Proposition 3.10

Every FRRLCS is a $FR\beta$ LCS.

Proof: Let A be FRRLCS. That is, $A = B \cap C$, where B is FR-ROS and C is FRLCS. By (viii) of Proposition 3.1, A is FRLCS. By Proposition 3.5, A is $FR\beta$ LCS.

Remark: Converse of the Proposition 3.6, Proposition 3.7, Proposition 3.8, Proposition 3.9 and Proposition 3.10 need not be true shown in the following example.

Example 3.3

Let $X = \{X_L, X_U\}$ be rough set where $X_L = \{a, b\}$ and $X_U = \{a, b, c\}$. Let $A = \{(a/.3, b/.4), (a/.4, b/.5, c/.6)\}$ and $B = \{(a/.3, b/.5), (a/.4, b/.5, c/.7)\}$ be any two FR-OS in X . The

family $T = \{\bar{0}, \bar{1}, A, B\}$ is a fuzzy rough topology on X . Then, the ordered pair (X, T) is fuzzy rough topological space. Now $E = A \cap \bar{A}$ is FRLCS (respectively FRSLCS, FRPLCS, FR α LCS, FR β LCS) but E is not FRRLCS in (X, T) .

Proposition 3.11

Every FR α LCS is a FRSLCS.

Proof: Let A be FR α LCS. That is, $A = B \cap C$ where B is FR- α -OS and C is FR- α -CS. By (vi) of Proposition 3.1, A is FRSLCS.

Remark: Converse of the Proposition 3.11 need not be true shown in the following example.

Example 3.4

Let $X = \{X_L, X_U\}$ be rough set where $X_L = \{a, b\}$ and $X_U = \{a, b, c\}$. Let $A = \{(a/.3, b/.4), (a/.4, b/.5, c/.6)\}$, $E = \{(a/.4, b/.5), (a/.4, b/.5, c/.1)\}$ and $F = \{(a/.3, b/.5), (a/.4, b/.5, c/.6)\}$ be any three fuzzy rough open sets in X . The family $T = \{\bar{0}, \bar{1}, A\}$ is a fuzzy rough topology on X . Then, the ordered pair (X, T) is fuzzy rough topological space. Now, $D = E \cap F$ is fuzzy rough semi locally closed but E is not FR α LCS in (X, T) .

Proposition 3.12

Every FRPLCS is a FR β LCS.

Proof: Let A be FRPLCS. That is, $A = B \cap C$ where B is FR-POS and C is FR-PCS. By (vii) of Proposition 3.1, A is FR β LCS.

Remark: Converse of the Proposition 3.12 need not be true shown in the following example.

Example 3.5

Let $X = \{X_L, X_U\}$ be rough set where $X_L = \{a, b\}$ and $X_U = \{a, b, c\}$. Let $A = \{(a/.3, b/.4), (a/.4, b/.5, c/.6)\}$, $B = \{(a/.3, b/.5), (a/.4, b/.5, c/.7)\}$ and $D = \{(a/.6, b/.6), (a/.6, b/.7, c/.1)\}$ any three FR-OS in X . The family $T = \{\bar{0}, \bar{1}, A, B\}$ is a fuzzy rough topology on X . Then, the ordered pair (X, T) is fuzzy rough topological space. Now, $E = D \cap \bar{D}$ is FR β LCS, but E is not FRPLCS in (X, T) .

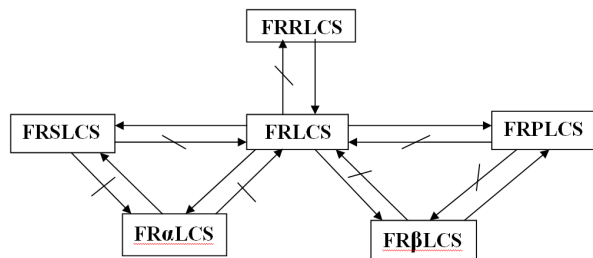


Figure 1: Interrelations between the fuzzy rough locally closed sets.

III. CONCLUSION

In this paper, authors introduced the concepts of fuzzy rough semi open sets, fuzzy rough pre-open sets, fuzzy rough α -open sets and fuzzy rough β -open sets and also discussed some of the properties and interrelations with examples. Authors also introduced some of the properties of fuzzy rough locally closed set, fuzzy rough semi locally closed, fuzzy rough pre locally closed, fuzzy rough α -locally closed set, fuzzy rough β -locally closed set and fuzzy rough regular locally closed set. The authors having further concepts under this study to extend this research to the next level.

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