

Homomorphism on Intuitionistic Fuzzy Subgroups-Some Aspects

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Abstract

In 1965, L.A.Zadeh introduced the concept of Fuzzy Set . As a generalization of the concept of Fuzzy Sets, Intuitionistic Fuzzy Sets were introduced by Atanassov in 1986, by taking into consideration of the grade of membership value and the grade of non-membership value for each element in the universe. Rosenfeld introduced the concept of Fuzzy Subgroups, which was generalized to the concept of Intuitionistic Fuzzy subgroups in a later period. In this paper some of the aspects of the Fuzzy Homomorphism on Intuitionistic Fuzzy Subgroups have been investigated.

INTRODUCTION

More generalizations of the ideas in the context of Fuzzy Sets have been made in various area. In this paper some of the results related to Fuzzy Subgroups have been generalized for Intuitionistic Fuzzy Subgroups and the focus is given in the area of Fuzzy Homomorphism.

1. Preliminaries

1.1 Definition: [1] Let X be a nonempty set. A Fuzzy Set A of X is an object of the form,

$A = \{(x, \mu_A(x))/x \in X\}$, where $\mu_A: X \rightarrow [0,1]$ defines the degree of membership of the element $x \in X$ and for every element $x \in X$, $0 \leq \mu_A(x) \leq 1$.

1.2 Definition: [2] Let X be a nonempty set. An Intuitionistic Fuzzy Set A of X is an object of the form, $A = \{(x, \mu_A(x), \nu_A(x))/x \in X\}$, where $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ and for every element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

1.3 Definition: An I.F.S, A on X which takes the values α and β as degree of Membership and degree of Non Membership at some point $x \in X$ where $0 \leq \alpha \leq 1, 0 \leq \beta \leq 1, 0 < \alpha + \beta \leq 1$ and takes the values 0 and 1 as degree of Membership and degree of Non Membership at all other points $y \in X, y \neq x$ is called an Intuitionistic Fuzzy point and is denoted by $x_{\alpha, \beta}$

1.4 Definition: [3] An I.F. Set A is said to be an I.F Subgroup of G if ,

- a) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y); \mu_A(x^{-1}) = \mu_A(x)$,and
- b) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y); \nu_A(x^{-1}) = \nu_A(x)$ for all $x, y \in G$

1.5 Definition: Let G be a group, and let A, B be Intuitionistic Fuzzy Subgroups of G . Using Zadeh's extension principle, define $A \circ B$ and A^{-1} as follows.

$$(A \circ B)(x) = (\mu_{A \circ B}(x), \nu_{A \circ B}(x)) , \text{ where } \mu_{A \circ B}(x) = \sup_{st=x} \min (\mu_A(s), \mu_B(t))$$

=0, if x can not be expressed as $x = st$, and

$$\nu_{A \circ B}(x) = \inf_{st=x} \max (\nu_A(s), \nu_B(t))$$

= 1, if x can not be expressed as $x = st$

$$A^{-1}(x) = (\mu_{A^{-1}}(x), \nu_{A^{-1}}(x)) \text{ where } \mu_{A^{-1}}(x) = \mu_A(x^{-1}), \nu_{A^{-1}}(x) = \nu_A(x^{-1})$$

Consequently, if $x_{\alpha,\beta}, y_{\gamma,\delta}$ are Intuitionistic Fuzzy points, then

$$x_{\alpha,\beta} \cdot y_{\gamma,\delta} = (x \cdot y)_{\alpha \wedge \gamma, \beta \vee \delta} \quad \text{and} \quad ((x_{\alpha,\beta})^{-1}) = (x^{-1})_{\alpha,\beta}$$

1.6. Definition: Let G be a group and let A be an Intuitionistic fuzzy Subgroup of G . Define ,
 $\tilde{A} = \{x_{\alpha,\beta} / \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta, x \in G\}$ where $\alpha \in [0, \mu_A(e)], \beta \in [\nu_A(e), 1]$

1.7 Definition: [4] Let A and B be I.F. Subgroups of G and G' respectively. A mapping, $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$ is said to be a Fuzzy Homomorphism if it preserves the group operation.

That is if $\tilde{f}(x_{\alpha,\beta} \cdot y_{\gamma,\delta}) = \tilde{f}(x_{\alpha,\beta}) \tilde{f}(y_{\gamma,\delta})$ for all $x_{\alpha,\beta}, y_{\gamma,\delta} \in \tilde{A}$

$\tilde{f}: \tilde{A} \rightarrow \tilde{B}$ is said to be a Fuzzy Isomorphism if it is a bijective fuzzy Homomorphism.

2. Properties

2.1 [6] Let A and B be I.F. Subgroups of G and G' respectively and let $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$ be a Fuzzy Homomorphism. Then a) $\text{supp } \tilde{f}(e_{\alpha,\beta}) = e'$ and $\text{supp } \tilde{f}(x_{\alpha,\beta}) = \text{supp } \tilde{f}(x_{\gamma,\delta})$ for all $\alpha, \gamma \in [0, \mu_A(e)]$ and $\beta, \delta \in [\nu_A(e), 1]$ and corresponding to the fuzzy Homomorphism, $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$, there exists a function $f: G \rightarrow G'$ and the restriction functions, $\phi_x: [0, \mu_A(e)] \rightarrow [0, \mu_B(e')]$ and $\psi_x: [\nu_A(e), 1] \rightarrow [\nu_B(e'), 1]$, such that $\tilde{f}(x_{\alpha,\beta}) = (f(x))_{\phi_x(\alpha), \psi_x(\beta)}$, for all $x_{\alpha,\beta} \in \tilde{A}$

2.2 [6] If A and B are I F Subgroups of G and G' respectively and $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$ be a fuzzy homomorphism, then \tilde{f} is order preserving.

2.3 Proposition: Let A and B be I.F. Subgroups of G and G' respectively. Then $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$, given by

$\tilde{f}(x_{\alpha,\beta}) = (f(x))_{\phi_x(\alpha),\psi_x(\beta)}$ is a Fuzzy homomorphism, if and only if the function $f: G \rightarrow G'$ is an ordinary homomorphism and the restriction functions, ϕ_x and ψ_x satisfy the equations,

$$\phi_{xy}(a \wedge b) = \phi_x(a) \wedge \phi_y(b) \text{ and } \psi_{xy}(a \vee b) = \psi_x(a) \vee \psi_y(b).$$

Proof: $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$ is a Fuzzy homomorphism

$$\Rightarrow \tilde{f}(x_{\alpha,\beta} \cdot y_{\gamma,\delta}) = \tilde{f}(x_{\alpha,\beta}) \tilde{f}(y_{\gamma,\delta}) \text{ for all } x_{\alpha,\beta}, y_{\gamma,\delta} \in \tilde{A}$$

$$\Rightarrow \tilde{f}((x \cdot y)_{\alpha \wedge \gamma, \beta \vee \delta}) = \tilde{f}(x_{\alpha,\beta}) \cdot \tilde{f}(y_{\gamma,\delta})$$

$$\Rightarrow (f(x \cdot y))_{\phi_{xy}(\alpha \wedge \gamma), \psi_{xy}(\beta \vee \delta)} = (f(x))_{\phi_x(\alpha), \psi_x(\beta)} \cdot (f(y))_{\phi_y(\gamma), \psi_y(\delta)}$$

$$\Rightarrow (f(x \cdot y))_{\phi_{xy}(\alpha \wedge \gamma), \psi_{xy}(\beta \vee \delta)} = (f(x) \cdot f(y))_{\phi_x(\alpha) \wedge \phi_y(\gamma), \psi_x(\beta) \vee \psi_y(\delta)}$$

$$\Rightarrow f(x \cdot y) = f(x) \cdot f(y) \text{ and } \phi_{xy}(\alpha \wedge \gamma) = \phi_x(\alpha) \wedge \phi_y(\gamma); \psi_{xy}(\beta \vee \delta) = \psi_x(\beta) \vee \psi_y(\delta)$$

Conversely let $f(x \cdot y) = f(x) \cdot f(y)$ and $\phi_{xy}(a \wedge b) = \phi_x(a) \wedge \phi_y(b)$; $\psi_{xy}(a \vee b) = \psi_x(a) \vee \psi_y(b)$.

$$\tilde{f}(x_{\alpha,\beta} \cdot y_{\gamma,\delta}) = \tilde{f}((x \cdot y)_{\alpha \wedge \gamma, \beta \vee \delta})$$

$$= (f(x \cdot y))_{\phi_{xy}(\alpha \wedge \gamma), \psi_{xy}(\beta \vee \delta)} = (f(x) \cdot f(y))_{\phi_x(\alpha) \wedge \phi_y(\gamma), \psi_x(\beta) \vee \psi_y(\delta)}$$

$$= (f(x))_{\phi_x(\alpha), \psi_x(\beta)} \cdot (f(y))_{\phi_y(\gamma), \psi_y(\delta)} = \tilde{f}(x_{\alpha,\beta}) \cdot \tilde{f}(y_{\gamma,\delta})$$

$\Rightarrow \tilde{f}$ is a Fuzzy homomorphism.

2.4 Proposition: Let A and B be I.F.Subgroups of G and G' respectively and let $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$, given by

$$\tilde{f}(x_{\alpha,\beta}) = (f(x))_{\phi_x(\alpha), \psi_x(\beta)} \text{ be a Fuzzy homomorphism. Then,}$$

a) the restriction functions, ϕ_x and ψ_x are order preserving.

b) ϕ_x is meet preserving and ψ_x is join preserving.

Proof: a) Since \tilde{f} is a Fuzzy homomorphism, $\phi_{xy}(a \wedge b) = \phi_x(a) \wedge \phi_y(b)$ and

$$\psi_{xy}(a \vee b) = \psi_x(a) \vee \psi_y(b).$$

Suppose $a \leq b \Rightarrow a \wedge b = a$

$$\Rightarrow \phi_{xe}(a \wedge b) = \phi_{xe}(b \wedge a) = \phi_x(b) \wedge \phi_e(a)$$

$$\Rightarrow \phi_x(a) = \phi_x(b) \wedge \phi_e(a)$$

$$\Rightarrow \phi_x(a) \leq \phi_x(b)$$

Also if $a \leq b$, then $a \vee b = b$

$$\Rightarrow \psi_{xe}(a \vee b) = \psi_x(a) \vee \psi_e(b).$$

$$\Rightarrow \psi_x(b) = \psi_x(a) \vee \psi_e(b).$$

$$\psi_x(a) \leq \psi_x(b)$$

$\Rightarrow \phi_x$ and ψ_x are order preserving.

$$c) \phi_x(a \wedge b) = \phi_{xe}(a \wedge b) = \phi_x(a) \wedge \phi_e(b)$$

$$\text{Also } \phi_x(a \wedge b) = \phi_{xe}(b \wedge a) = \phi_x(b) \wedge \phi_e(a)$$

$$\begin{aligned} \Rightarrow \phi_x(a \wedge b) &= \phi_x(a) \wedge \phi_e(b) \wedge \phi_x(b) \wedge \phi_e(a) \\ &= (\phi_x(a) \wedge \phi_e(a)) \wedge (\phi_x(b) \wedge \phi_e(b)) \\ &= \phi_x(a) \wedge \phi_x(b) \end{aligned}$$

$\Rightarrow \phi_x$ is meet preserving.

$$\text{Similarly, } \psi_x(a \vee b) = \psi_{xe}(a \vee b) = \psi_x(a) \vee \psi_e(b)$$

$$\text{Also } \psi_x(a \vee b) = \psi_{xe}(b \vee a) = \psi_x(b) \vee \psi_e(a)$$

$$\Rightarrow \psi_x(a \vee b) = \psi_x(a) \vee \psi_e(b) \vee \psi_x b \vee \psi_e(a)$$

$$\Rightarrow \psi_x(a \vee b) = (\psi_x(a) \vee \psi_e(a)) \vee (\psi_x b \vee \psi_e(b))$$

$$\Rightarrow \psi_x(a \vee b) = \psi_x(a) \vee (\psi_x b)$$

$\Rightarrow \psi_x$ is join preserving.

2.5 Proposition: Let A and B be I.F.Subgroups of G and G' respectively and let $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$, given by, $\tilde{f}(x_{\alpha,\beta}) = (f(x))_{\phi_x(\alpha),\psi_x(\beta)}$ be a Fuzzy homomorphism. Then, \tilde{f} is surjective if and only if the function f and the restriction functions ϕ_x and ψ_x corresponding to each element $x \in G$ are all surjective. Also if each of the restriction functions, ϕ_x and ψ_x are identity functions, then \tilde{f} is injective if and only if f is injective.

Proof: Suppose \tilde{f} is surjective.

Let $y \in G'$ and let $\mu_B(y) = \gamma; \nu_B(y) = \delta$

$$y_{\gamma,\delta} \in \tilde{B}$$

Since \tilde{f} is surjective, $\exists x_{\alpha,\beta} \in \tilde{A}$, such that $\tilde{f}(x_{\alpha,\beta}) = y_{\gamma,\delta}$

$$\Rightarrow (f(x))_{\phi_x(\alpha),\psi_x(\beta)} = y_{\gamma,\delta}$$

$$\Rightarrow \phi_x(\alpha) = \gamma \text{ and } \psi_x(\beta) = \delta$$

$\Rightarrow \phi_x$ and ψ_x are surjective.

Conversely suppose that f, ϕ_x and ψ_x are all surjective.

Then, for $y_{\gamma,\delta} \in \tilde{B}$, \exists elements x, α, β such that $f(x) = y, \phi_x(\alpha) = \gamma$ and $\psi_x(\beta) = \delta$

$$\text{Also } \tilde{f}(x_{\alpha,\beta}) = (f(x))_{\phi_x(\alpha),\psi_x(\beta)} = y_{\gamma,\delta}$$

$\Rightarrow \tilde{f}$ is surjective

Now let \tilde{f} be injective and let the restriction functions ϕ_x and ψ_x be identity functions.

Let $f(x) = f(y)$

$$(f(x))_{\alpha,\beta} = (f(y))_{\alpha,\beta}$$

$$\Rightarrow (f(x))_{\phi_x(\alpha),\psi_x(\beta)} = (f(y))_{\phi_y(\alpha),\psi_y(\beta)}$$

$$\Rightarrow \tilde{f}(x_{\alpha,\beta}) = \tilde{f}(y_{\alpha,\beta})$$

$$\Rightarrow x_{\alpha,\beta} = y_{\alpha,\beta}$$

$$\Rightarrow x = y$$

$\Rightarrow f$ is injective.

Conversely suppose that f is injective.

$$\text{Let } \tilde{f}(x_{\alpha,\beta}) = \tilde{f}(y_{\gamma,\delta})$$

$$\Rightarrow (f(x))_{\phi_x(\alpha),\psi_x(\beta)} = (f(y))_{\phi_y(\gamma),\psi_y(\delta)}$$

$$\Rightarrow (f(x))_{\alpha,\beta} = (f(y))_{\gamma,\delta}$$

$$\Rightarrow f(x) = f(y); \alpha = \gamma, \beta = \delta$$

$$\Rightarrow x = y; \alpha = \gamma; \beta = \delta$$

$$\Rightarrow x_{\alpha,\beta} = y_{\gamma,\delta}$$

$\Rightarrow \tilde{f}$ is injective

2.6 Proposition: If A and B are I.F.Subgroups of G and G' respectively and if $\tilde{f}: \tilde{A} \rightarrow \tilde{B}$, is given by, $\tilde{f}(x_{\alpha,\beta}) = (f(x))_{\phi_x(\alpha),\psi_x(\beta)}$. Then, \tilde{f} is a Fuzzy isomorphism if and only if $f: G \rightarrow G'$

is an ordinary isomorphism and each of the restriction functions ϕ_x and ψ_x are identity functions.

Proof: Result follows by the above proposition.

3.Conclusion: Relative to I.F.Sets and I.F Subgroups, some of the aspects of the Fuzzy homomorphism has been investigated . More indications of the aspects of Fuzzy Homomorphism in the context of fuzzy Subgroups can be applied for I.F subgroups also .

References

- [1] L.A Zadeh , “Fuzzy sets”, Information and Control 8,(1965),338-353
- [2] K.T. Atanassov , “Intuitionistic fuzzy sets”, Fuzzy Sets and Systems 20(1986), NO-1,87-96
- [3] Sharma P.K (2011), “Intuitionistic fuzzy Groups”, International Journal of Data Ware Housing & Mining (IIJDWM), Vol.1,Issue 1,86-94
- [4] Fang Jin-xuan , “Fuzzy homomorphism and fuzzy isomorphism”, Fuzzy Sets and systems 63 (1994) 237-242
- [5] P.K. Sharma, “Homomorphism of intuitionistic Fuzzy Groups”, International Mathematical Forum, Vol.6 ,2011,no.64,3169-3178
- [6] Sheena K P, “Homomorphism-A Study on Intuitionistic Fuzzy Subgroups”, Indian journal of Natural Sciences(Communicated)