

Teaching design of prism concept from HPM perspective

Yumei Huang, Liang Fang*

College of Mathematics and Statistics, Taishan University, Tai'an, China

*Email:fangliang3@163.com

Abstract—The prism concept is the content of the structure of spatial geometry in volume 2 of the high school curriculum, which is an extension of the related content of solid figures. The study of prism concept can cultivate students' mathematical core literacy ability, such as geometric intuition and mathematical abstraction, and lay a good foundation for other geometry to be studied later. To design the teaching content of this lesson from the perspective of HPM, use relevant historical materials, the design problems and the appropriate operation process, let the student through the actual operation and in particular the process to solve the problem, understand the definition of prismatic emergence and development process, deeply understand the concept of prism can core in the development of students' mathematical literacy and subtly mathematics subject moral education.

Index Terms— HPM teaching, History of mathematics, prismatic concept, refactoring, core literacy.

I. INTRODUCTION

HPM is short for International Study Group on the Relations between History and Pedagogy of Mathematics. Now we also call the relationship between the history of mathematics and mathematics education itself HPM. HPM teaching mainly refers to the teaching of integrating mathematics history, mathematical methods and mathematical ideas into mathematics classroom teaching [1-3].

In the past 50 years, scholars and teachers have made more and more research on the integration of mathematics history in mathematics education. HPM teaching can help students understand mathematics more easily and increase the interest of mathematics learning. HPM teaching can enable students to deeply understand the evolution process of mathematical concepts, calculation methods and expressions, and better understand the changes of mathematical language and mathematical forms over time. The infiltration of mathematics history into mathematics teaching can promote students' profound understanding of mathematics and stimulate students' creativity.

East China normal university mathematics professor Xiaoqin Wang, on the basis of the existing research on HPM: the history of mathematics and mathematics education in the book, according to the history of mathematics and mathematics teaching content correlation degree, the level of

use of the history of mathematics in mathematics teaching, the original ornament, additional, replication, and consolidated and improved in the division of compliant, adjusting for: Add-on type, copy type, adapt type, reconstruct type four types. In China's current mathematics teaching, a large part of the use of Professor Xiaoqin Wang proposed this classification[2].

In this paper, from the perspective of HPM and based on the history of the formation of prism concept, mathematics history is integrated into the teaching of prism concept, which can deepen students' understanding of prism concept, develop students' mathematical core literacy, and be conducive to the implementation of mathematical discipline moral education [3].

II. TEACHING OBJECTIVES AND DESIGN OF PRISM CONCEPT FROM HPM PERSPECTIVE

When learning spatial geometry, students often have some problems, such as not understanding the concept thoroughly, lack of spatial imagination ability, observation of graphics is not comprehensive and so on. A review of the history of the formation of prism concept is consistent with students' cognitive law, which can guide students to analyze the rigor of definition with dialectical thinking through the process of question inquiry in teaching design, and enhance their confidence in learning geometry knowledge through understanding the development and perfection of prism definition.

A. The teaching objectives of the prism concept

First of all, students should intuitively understand the prism, and deeply understand the concept of prism, grasp the characteristics of straight prism, regular prism, can accurately use the concept of prism to judge different forms of prism. Then, can use the three-dimensional space "point, line, surface", through the quantitative relationship and position relationship of polyhedron research, through the research can be found and summarized various similarities and differences. At the same time, through the integration of prism concept related mathematical history data, let students understand the prism definition development process, grasp the development and rigor of mathematical concepts, encourage students to dare to ask

questions, dare to question, have the spirit of exploring the truth.

B. Teaching design of prism concept

Using interesting magnetic plates for convenient construction, let students more intuitive understanding of polyhedron. Then, the students are presented with various shapes of objects in life through pictures, so as to guide the students to ignore other factors of the objects, and only consider the size and shape of the objects to abstract into spatial geometry.

Guides the student to observe "point, line, face" in the geometry relationship, let the students try to give the definition of prismatic description, by comparison with the prism defined in the history of the students, analysis the vulnerability of definition given by the students, illustrate the prismatic concept appear problem, in the history of examples of concave polyhedron and convex polyhedron counterexample to analysis of definition, to deepen the understanding of students. Summing up and giving the precise definition of prism, and using the definition to judge whether the graph is a prism, to consolidate knowledge and expand thinking.

In course design, by observing the geometric features of prism, we summarize the common characters and seek for problems until we get accurate expressions. This is a process of abstract and accurate extension of concept. Students unknowingly experienced the emergence and development of the prism concept in the classroom.

The use of popular magnetic film toys in the classroom can make students better operation and experience; The process of students defining the prism by themselves increases the participation of students, and in the process of experiencing the inquiry, increases the fun and improves the sense of achievement. Let the students carefully and comprehensively observe the concrete objects, abstract the geometric model of prism from different angles, and analyze the quantitative relationship and position relationship in the graph, so as to cultivate the students' mathematical literacy of intuitive imagination and mathematical abstraction.

III. IMPLEMENTATION OF TEACHING CASES

(1) Operate observation and introduce new knowledge

Before class, arrange the students to build a closed shape with magnetic sheets. Ask the students to observe the different closed bodies in groups and summarize the common features of each face of the shape. The students find that each face is a polygon. Use multimedia to show some related pictures, let the students observe the geometry intuitively, can let the students have a better understanding of the space graphics.

The teacher concluded that in mathematics, a closed

geometry surrounded by a plane polygon (including a triangle) is called a polyhedron, each plane polygon that forms a polyhedron is called the face of the polyhedron, the common edge of each adjacent face is called the edge of the polyhedron, and the intersection point of each edge is the vertex of the polyhedron.

(2) Choose a few visual mock-ups and draw pictures of them, such as the following:

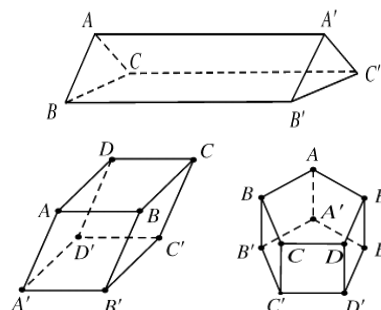


Fig. 1 Three geometric models

Guide the students to start from the three basic quantities of "point, line and plane", analyze the relationship between quantity and position, sum up the common characters, and complete the contents in the table.

Table 1 Quantity relation and location relation

Basic amount	Quantitative relation	Relation of position
point	The number of vertices is uncertain	There is no particular location
line	The number of edges is uncertain.	1.Relative edges of two congruent polygons are parallel. 2.The edges in the other planes are parallel to each other
plane	1.The number of planes is uncertain. 2.Two opposing polygons are congruent.	1.Two congruent polygon planes are parallel. 2.All the other planes are parallelograms.

After that, let the students try to abstract the definition of polyhedron according to the common character through the number and position relationship of "point, line and plane" in the table.Guide the students to summarize and describe, students get some similar description slightly different concepts.

(3) Explore the concept of prism in the context of history

In the third century BC, a famous Greek mathematician

named Euclid first defined a prism in this way in Euclid's Elements: "A prism is a solid figure consisting of a number of planes in which two faces are opposite, equal, similar and parallel, and the other faces are parallelograms." Euclid is a very authoritative mathematician, and Euclid's Elements is a very professional work in the history of mathematics. This definition has been passed down for more than twenty centuries [4].

Let the students feel whether this definition is somewhat similar to the abstract definition of many classmates, let the students think about what is not rigorous in this expression, let the students understand that the process of exploring the truth is not accomplished overnight. The geometry which does not meet Euclid's definition is splited by using two four-prism which students put together with magnetic plates, and a counterexample of Euclid's definition is obtained [5-7].

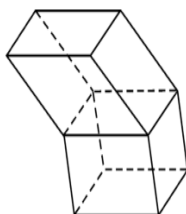


Fig. 2 A counterexample of Euclid's definition

In Euclid's definition, the two opposite faces of a prism are congruent polygons, and the faces outside these two faces are parallelograms, but their quantitative relationship is not explained. In quantitative terms, a n prism ($n > 2$, $n \in \mathbb{Z}$) has $n+2$ planes, $3n$ edges, and $2n$ vertexes. Thus, a four-prism should have 6 faces, whereas the polyhedron shown as a counterexample has 11 faces, so Figure 2 is a counterexample of Euclid's definition, called a concave polyhedron. Let n be the number of edges of parallel congruent polygons, and further summarize and describe the number and position relations of prisms, and the following table can be obtained.

Table 2 A quantitative relation in a prism

Basic amount	Quantitative relation
point	$2n$ vertexes
line	$3n$ edges
plane	1. $n+2$ planes 2. Two opposite congruent n polygons

In 1916, American mathematicians John C. Stone and James F. Millis again gave a counter-example of Euclid's definition. Stone gives a counter example of dodecahedron convex polyhedron, which is more common than concave polyhedron.

Below is a counterexample Stone gave at the time.

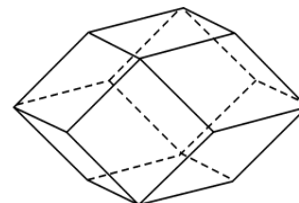


Fig. 3 A counter example of a convex polyhedron

This counter example can be made with 8 diamonds and 4 squares of magnetic plates. Since there is no restriction on the number of faces in Euclid's definition, many counter examples can be constructed by expanding the number of faces in a similar way.

Students gain a deep understanding of the definition of prism through observation and summary, in conjunction with the development of prism definition. In general, two planes are parallel to each other, the remaining planes are quadrilaterals, and the common sides of adjacent quadrilaterals are parallel to each other. Such a polyhedron is called a prism. Two surfaces that are parallel to each other are called the base plane of a prism. The remaining faces are called the sides of the prism, and the common point between the sides and the base is called the apex of the prism. The common edge of the adjacent sides is called the lateral edge of the prism, and the distance between the two bases is the height of the prism.

(4) Example explanation for further understanding

Give some geometry, let the students intuitively determine whether it is prism according to the definition, such as the geometry below.

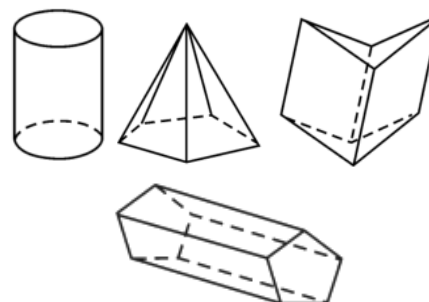


Fig. 4 Four geometries

The formation of prism concept has gone through the process from entity to definition, from individuality to commonness, from figure to abstraction, from worship to questioning, from error to correctness, from uniqueness to contention of a hundred schools of thought. The development of mathematical truth is often tortuous and repeated, but the steps of exploring the truth have never stopped, through the efforts of generation after generation, has been constantly improving.

Through the practical operation, summarize the abstract, in the class, let students try to define the prism, compare

the definition of Euclid, deepen the understanding of the prism, have a clearer concept. In the process of constantly proposing various expressions of the definition of prism, and discussing and improving the definition, the experience of correcting errors shows the charm of mathematics.

Ask students to expand the examples of dodecahedron given in class and study how to use magnetic plates to build similar geometry, such as hexahedron.

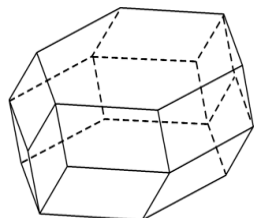


Fig. 5 A counter example of a hexahedron

(5) Experience history and summarize thinking

Through the history of prism definition, students can experience the twists and turns of the development of mathematical truth, and understand that the study of mathematicians can also be imperfect. Let the students realize that many conclusions are the further exploration on the basis of predecessors. Let the students build up confidence that the younger generation through hard work in the future may also supplement the existing theory, or develop new mathematical theory. Let students understand that the theory in the development of the time will always go through the process of continuous development of trial and error, we should have their own thinking, not blindly follow nor stubborn, to seek truth from facts, to explore the truth.

IV. CONCLUSION

In the teaching design of prism concept in this paper, there are three kinds of application to the history of mathematics: additive, adaptive and reconstruction. Ask students to come up with their own definition of prism, compared with the concept of prism by historical mathematicians, is additive. Ask students to analyze the historical definition of prism, which belongs to the compliant form. On the whole, through the evolution and development of the definition of prism in the history of mathematics, we reconstruct this process in class, which belongs to the reconstruction.

In this case, the historical story of the imperfect prism definition given by the mathematician Euclid Narrows the distance between students and mathematicians and mathematical knowledge, and enables students to learn mathematical knowledge with emotion and enthusiasm. At the same time, the mistakes of mathematicians in history also give students a warning, which makes them

understand that in study and research, they should not blindly worship authority, but should have the courage to question and the spirit to explore the truth. The process of revising the definition of prism allows students to experience the development of mathematics and establish a dynamic view of mathematics. Students will be tolerant of the "mistakes" of Euclid and appreciate his great contribution to geometry. Through the process of building counter examples and abstracting summary constantly, students gain a strong sense of achievement and self-confidence, and realize the effect of moral education for students.

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Author's biography with Photo



Yumei Huang is a lecturer at Taishan University. She obtained her master's degree from Shandong University of Science and Technology in July, 2008. Her research interests are in the areas of applied mathematics and mathematics education in recent years. email id: huangyumei125@163.com



Liang Fang was born in December 1970 in Feixian County, Linyi City, Shandong province, China. He is a professor at Taishan University. He obtained his PhD from Shanghai Jiaotong University in June, 2010. His research interests are in the areas of cone optimizations, numerical analysis, and complementarity problems.