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Case teaching of Bayes formula combined with mathematical modeling idea

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Abstract—*The paper from the new explanation, knowledge application, the development of class three teaching links to mathematical modeling thought into the Bayes formula in case teaching, explore the mathematical modeling thought into the concrete implementation strategy of the whole process of teaching, arouse the students' interest in using mathematical ideas to solve practical problems, improve the students' ability of mathematical modeling.*

Keywords—Bayes formula, mathematical modeling, case teaching

I. INTRODUCTION

For nearly half a century and more, with the rapid development of computer technology, the application of mathematics is not only in the field of engineering technology, natural science is playing an increasingly important role, but with an unprecedented breadth and depth to the economy, management, finance, biology, medicine, environment, geological, population, transportation, and other areas of the new penetration, the so-called mathematics technology has become an important part of modern high-tech. When a practical problem needs to be analyzed and studied from a quantitative point of view, people should establish a mathematical model with mathematical symbols and languages on the basis of in-depth investigation, understanding of object information, making simplified assumptions and analyzing internal laws [1]. Mathematical modeling is to establish a mathematical model according to the actual problem, to solve the mathematical model, and then solve the actual problem according to the results. This paper introduces the idea of mathematical modeling into the teaching reform of probability theory and mathematical statistics, aiming at enhancing the ability of college students to solve practical problems skillfully with mathematical knowledge, cultivating students' interest in learning, and further cultivating students' innovation ability.

II. BAYES FORMULA AND ITS GENERALIZATION

A. *Bayes formula and its principle*

Bayes formula: Let $A_1, A_2, \cdots, A_n$ be the division of the sample space S, $A_1, A_2, \cdots, A_n$ is incompatible and their sum is S, B is the target event,if $P(B)>0$, $P(A_i)>0\,(i=1,2,\cdots,n)$, we have

$$P(A_i|B)=\frac{P(A_i)P(B|A_i)}{\sum_{i=1}^{n}P(A_i)P(B|A_i)},i=1,2,\cdots,n.$$

Bayes formula provides an effective means to modify the original judgment by using the collected information. Before sampling, the economic subject to various assumptions have a judge (prior probability), about the prior probability distribution, and usually can be determined according to the experience of the economic subject judgment (when no information, generally assume that the prior probability is the same), more complex precise available include the maximum entropy technology or marginal distribution density and the principle of mutual information method to determine the prior probability distribution [2].

B. *Generalization and proof of Bayes formula*

Suppose that when there are no less than two random processes in the experiment, complete event groups are established respectively in each experiment process affecting the target event, and Bayes formula can be further extended.

Generalization theorem of Bayes formula : $A_i\,(i=1,2,\cdots,n.)$ and $B_j\,(j=1,2,\cdots,m.)$ are the division of sample space in two experiments successively. C is the target event, if $P(C)>0$, then

$$P(A_i)>0, P(B_j)>0, P(A_iB_j)>0,$$
$$(i=1,2,\cdots,n;\ j=1,2,\cdots,m),$$

so, we have

$$P(A_i|C)=\frac{\sum_{j=1}^{m}P(A_i)P(B_j|A_i)P(C|A_iB_j)}{P(C)}, \quad (1)$$
$$(i=1,2,\cdots,n).$$

$$P(B_j|C)=\frac{\sum_{i=1}^{n}P(A_i)P(B_j|A_i)P(C|B_j)}{P(C)}, \quad (2)$$
$$(j=1,2,\cdots,m).$$

$$P(A_iB_j|C)=\frac{P(A_i)P(B_j|A_i)P(C|A_iB_j)}{P(C)}, \quad (3)$$
$$(i=1,2,\cdots,n;\ j=1,2,\cdots,m).$$

Prove that (1)

$$P(A_i|C)=\frac{P(A_iC)}{P(C)}=\frac{\sum_{j=1}^{m}P(A_iB_jC)}{P(C)}$$
$$=\frac{\sum_{j=1}^{m}P(A_i)P(B_j|A_i)P(C|A_iB_j)}{P(C)},(i=1,2,\cdots,n).$$

It can be proved in the same way (2) and (3).

C. Case and summary of Bayes formula

Case 1:A villa has been burgled twice in the past 20 years. The owner of the villa has a dog, and the dog barks three times a week in the evening when there is no burglar. The probability of the dog barks when the burglar intruders is 0.9.

Let's assume that the event C is dog barking at night, and the event D is burglar intruding, given the known conditions,

$$P(D)=\frac{2}{20\times 365}=\frac{1}{3650},\ P(C|D)=0.9,\ P(C|\overline{D})=\frac{3}{7}.$$

It's easy to follow the formula:

$$P(D|C)=\frac{P(D)P(C|D)}{P(D)P(C|D)+P(\overline{D})P(C|\overline{D})}$$
$$=\frac{\frac{1}{3650}\times 0.9}{\frac{1}{3650}\times 0.9+\frac{3649}{3650}\times\frac{3}{7}}=0.000575.$$

That is, the probability of an invasion occurring while the dog barks is 0.000575.

In the classical Bayes formula, random sequence of events is required to be incompatible. If this condition is weakened, a generalized Bayes formula can be given, which can be directly calculated regardless of compatibility. From the form of the formula, increase the flexibility of the formula. For example, in the classical Bayesian formula, the samples are discrete, but in the actual calculation, it is not very practical when complex events are encountered. At this time, the total probability formula can be extended to the case of random variables [3]. Of course, random variables can be discrete, or they can be continuous, or they can be mixed, so you can use the distribution law to solve the problem. Starting from the computational assistance of the formula, the popularization of the formula is used in the improvement of risk model, risk calculation and risk process analysis.

III. INTRODUCE NEW KNOWLEDGE THROUGH CASE TEACHING BY COMBINING IDEAS AND METHODS OF MATHEMATICAL MODELING

The novel corona-virus outbreak has made wearing face masks, a common precaution in China, a more difficult one to implement in the U.S. and Europe. Use probabilities to explain the necessity of wearing a mask. Analysis (actual problem math) : In a high-risk area, assuming that the residents of the area maintain a safe social distance and eye protection, if a person is known to be infected with COVID-19, the probability that he or she will be infected because he or she is not wearing a mask is asked.If event A means "residents are infected with novel corona-virus", event B means "residents wear masks", then the math problem is $P(\overline{B}|A)$.

A. Model assumes

a.Suppose that in an area where masks are preferred, 95% of people are used to wearing masks and 5% are not used to wearing masks. b.Suppose that in an area where masks are not preferred, 5% of the people are used to wearing masks and 95% are not used to wearing masks.c.It is assumed that in high-risk areas, the probability of infection is 3.1% when wearing masks, and 17.4% when not wearing masks [4].

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B. Mathematical modeling

According to the conditional probability formula, $P(\overline{B}|A)=\frac{P(A\overline{B})}{P(A)}$.For the numerator, we know by the product formula that $P(A\overline{B})=P(\overline{B})P(A|\overline{B})$. For the denominator $P(A)$, it can be seen from the question that the result of "residents infected with COVID-19" is likely to occur in two cases, excluding other cases. They are respectively in case 1 -- residents wearing masks, or in case 2 -- residents not wearing masks. Therefore, "resident infected new coronet" can include two situations: resident infected while wearing a mask, or resident infected when not wearing a mask, i.e $A=AB+A\overline{B}$, so $P(A)=P(AB+A\overline{B})$.

Because AB is incompatible with $A\overline{B}$, according to the finite additivity of probability, $P(A)=P(AB)+P(A\overline{B})$.

Use the multiplication formula,

$P(AB)=P(B)P(A|B)$, $P(A\overline{B})=P(\overline{B})P(A|\overline{B})$,we have

$$P(A)=P(B)P(A|B)+P(\overline{B})P(A|\overline{B}).$$

So this is the total probability formula that we've learned. Substituting the numerator and denominator respectively, we can get

$$P(\overline{B}|A)=\frac{P(\overline{B})P(A|\overline{B})}{P(B)P(A|B)+P(\overline{B})P(A|\overline{B})}.$$

C. Model calculation

If you are in an area where masks are not preferred, it is assumed that, $P(B)=0.05$, $P(\overline{B})=0.95$, $P(A|B)=0.031$, $P(A|\overline{B})=0.174$. The probability is

$$P(\overline{B}|A)=\frac{0.95\times0.174}{0.05\times0.031+0.95\times0.174}\approx0.99071.$$

In order to facilitate students' comparison, students can be guided to calculate by themselves "if a person is known to be infected with COVID-19, ask for the probability of his being infected if he is wearing a mask", i.e., calculate $P(B|A)$.

In a region that prefers to wear masks, it is assumed that,

$P(B)=0.95$, $P(\overline{B})=0.05$, $P(A|B)=0.031$, $P(A|\overline{B})=0.174$ .The probability is:

$$P(\overline{B}|A)=\frac{0.95\times0.031}{0.95\times0.031+0.05\times0.174}\approx0.77195.$$

D. Model application

Through calculation, it is found that if a resident is unfortunately infected with COVID-19, the probability of being infected without wearing a mask is as high as that in a region that prefers wearing a mask. This result fully verifies the epidemic prevention situation faced by western countries and China in preventing Covid-19, and the result is convincing. At the same time, it explains the necessity of wearing masks for epidemic prevention from the perspective of probability theory. Bayesian formula, which is very important in probability theory, is used in the process of solving. Teachers can continue to guide students to summarize the use conditions, applicable question types (fruit-holding reasons) and extended forms (which can be extended to N reasons) of Bayesian formula. While solving practical problems, through the mathematical modeling thought, the theme content of this course-Bayesian formula is derived step by step from the shallow to the deep, and students feel that the formula is given reasonably, thus avoiding the dilemma that students feel the learning of knowledge points such as rootless trees and passive water when explaining the formula directly [5].

IV. KNOWLEDGE CONSOLIDATION AND DEVELOPMENT

Case3 Using probability knowledge, this paper explains how the villagers' trust in the shepherd boy declined in the fable of "Wolf Comes".

A. Model assumptions

①Assume that the probability of a child being an honest child is 0.9; ② Assume that the probability that an honest child will lie is 0.1; ③ Assume that the probability that a dishonest child will lie is 0.6.

B. Establishing the model

Event A means that children lie, event B means that children are honest, and event $\overline{B}$ means that children are dishonest. Bayes formula is applied:

$$P(B|A)=\frac{P(B)P(A|B)}{P(B)P(A|B)+P(\overline{B})P(A|\overline{B})}.$$

C. Model calculation

According to the hypothesis,

$$P(B)=0.9, P(\overline{B})=0.1,\ \ P(A|\overline{B})=0.6,\ \ P(A|B)=0.1.$$

After the shepherd boy lied for the first time, the villagers trusted him as follows: $P(B|A)=\frac{0.9\times0.1}{0.9\times0.1+0.1\times0.6}=0.6$. After the sheep herder lied for the first time, in the eyes of the villagers, the probability that the child is honest is reduced, that is, it can be considered at this time $P(B)=0.6$, $P(\overline{B})=0.4$. Continue to calculate, the villagers' trust in the shepherd boy after lying for the second time: $P(B|A)=\frac{0.6\times0.1}{0.6\times0.1+0.4\times0.6}=0.2$.

D. Model application

After the shepherd boy lied twice, the villagers' trust in the shepherd boy declined from the initial to 0, so the story ended miserably, and the shepherd boy paid the price of his life for his repeated lies. This example is an application case of Bayesian formula. Through mathematical modeling and analysis of the fable "Wolf Comes", students can not only deeply understand the profound connotation of "being honest and being a man" contained in the fable, but also help students master the conditions and applicable questions of Bayesian formula, and cultivate students' ability to solve practical problems by using their mathematical knowledge, which greatly stimulates their interest in learning [6].

Case4 The key algorithm core of many artificial intelligence phenomena, such as spam recognition, smart phone automatic translation, speech recognition, etc., is Bayesian formula. Each group is invited to discuss how Bayesian formula is applied to modeling these practical problems in combination with the ideas and methods of mathematical modeling, and submit a report to the group [7]. This is an open homework. After class, students can consult materials, discuss in groups, cooperate in teams, and complete a research report with the ideas and methods of mathematical modeling.

V. Conclusion

Bayesian formula plays a very wide role in solving mathematical models, and it is a tool that is often used in mathematical models. With the rapid development of society, decision-makers must comprehensively examine the past information and present situation to make comprehensive judgments, and decision probability analysis is increasingly showing its importance. Among them, Bayesian formula is mainly used to deal with prior probability and posterior probability, which is an important tool for decision-making [8]. Teachers consciously integrate the ideas and methods of mathematical modeling in all aspects of classroom teaching to carry out case teaching, combine theoretical knowledge with practical problems, make students fully aware of the practical application of the course, and make the course study from useless to useful; Let students gradually understand the essence of mathematical modeling, master the basic methods of mathematical modeling, cultivate students' ability to solve practical problems and innovate, stimulate their interest in learning, and make course learning interesting from boring; Finally, the students' understanding of the curriculum knowledge is deepened, which improves the learning efficiency and makes it difficult to learn the curriculum from difficulties. With the deepening of teaching activities, how to integrate the ideas and methods of mathematical modeling more effectively and continuously improve the teaching quality needs further discussion and improvement.

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