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Research on the Cultivation of Students' Mathematical Abstract Thinking Ability

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***Abstract*—Mathematical thinking ability is the core of mathematical ability, and mathematical abstract thinking ability is an important part of mathematical thinking ability. Like thinking ability of training students' smoking is one of the aims of mathematics education, is one of the important basis, students can learn to use math well is a basic requirement to realize the objective of mathematics teaching, also help students form a more perfect structure of thinking, in this paper, the mathematical abstract thinking concepts and related theories are analyzed, aiming at the problems in students' learning mathematics abstract thinking, This paper probes into the cultivation of students' abstract ability in mathematics teaching, and puts forward the theory and teaching measures of cultivating students' abstract thinking ability in mathematics, in order to provide certain theoretical and practical reference for improving students' core quality in mathematics abstraction.**

***Index Terms*—Core literacy, Mathematical thinking, the abstract thinking ability, Training, Mathematics core literacy**

I. INTRODUCTION

Mathematics is a science that studies quantity and form and their relations. Mathematics plays a unique role in promoting the development of individual intelligence and forming rational thinking. Mathematical thinking ability is one of the core contents of mathematical ability. Mathematical thinking ability is an ability to abstract, think and judge the quantitative relationship and spatial form of objective things by induction, abstraction and generalization, taking the mathematical research object as the carrier. This ability is indispensable for improving human rational thinking and constructing mathematical literacy [1-2].

Mathematical thinking has three characteristics: abstraction, logic and formalization. Mathematical concepts are highly abstract and generalized according to the essential attributes of objects. Mathematics is a deductive science, closely linked with logic, common development, mathematical thinking has a strong logic; The formation of mathematics is accompanied by the development and research of the symbolic system. The symbolic and formalization of mathematics is a manifestation of rigorous mathematical thinking. Abstract thinking ability of mathematics is an important part of mathematical thinking ability, which is the key to improve students' core literacy level of mathematics abstraction [3-4].

II. THE CONCEPT AND FUNCTION OF MATHEMATICAL ABSTRACT THINKING

A. The concept of mathematical abstract thought

Abstract mathematical thinking is a kind of mathematical thinking, which is based on mathematical objects or mathematical contents, and extracts the common and essential attributes or characteristics of similar things to form new things. This new thing can be a concept or a method. The basic method of mathematical abstract thinking is similar to the thinking method of natural science, such as "observation, experiment, analogy, induction", and similar to the thinking method of social science, such as "refutation, speculation, imagination, intuition". For example, human beings abstracted the concept of number from the initial number of objects, and a large number of mathematical contents or phenomena were expressed in the mathematical sign language after abstraction..

B. The function of mathematical abstract thought

Abstract thinking plays an important role in mathematical proof. In learning, often in the process of proving some mathematical theorems or laws, encountered difficulties, some difficulties through the role of logical thinking can be overcome, some difficulties are difficult to cross. The reason why people can't solve these difficulties lies in the fact that abstract thinking has not fully played its role and the understanding of abstract thinking has not reached enough depth. Abstract comes from concrete, and to understand abstract mathematical proofs. It requires a concrete knowledge of sensibility or thinking in people's minds. No matter at what stage, mathematical abstract thinking is the interaction between weak abstraction and strong abstraction.

The American mathematician M. Klein put it this way: "Premature abstractions fall to the deaf ears, whether they belong to the mathematicians or to the students." That is to say, the development of abstract thinking has its inherent regularity, and people should follow its law to learn [5-8].

Abstract thinking plays an important role in the extension of mathematics. In the history of mathematics, abstraction and generalization have alternated. Teachers should pay attention to cultivating students' abstract thinking, in order to better realize the teaching of mathematical knowledge and help students to learn mathematics more easily. Abstract thinking

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not only involves the creation of mathematical objects, but also affects the use of mathematical methods. The mathematical method here mainly refers to the processing methods of mathematical objects, including mathematical analysis, proof and extension.

The analysis method here mainly refers to grasping the essence of the problem through analysis, transforming the problem into a form, so as to achieve the purpose of simplifying into easy and complex into simple. If more complex problems are encountered, it is also necessary to decompose the converted problems into various components or several possible steps, and then solve the problems of each part, so as to solve all the problems. Using abstract thinking in mathematical analysis can help students grasp the essence of the problem quickly and find the answer to the problem smoothly.

Generally speaking, abstract thinking plays an indispensable role in the process of mathematical analysis, proof and extension. Abstract thinking is the product of the left brain. Therefore, teachers should admit that mathematical objects are the products of abstract thinking and all come from the left brain. However, mathematical thinking also involves the right brain, such as speculation, imagination, intuition, reasonable reasoning and so on, which are closely related to the right brain thinking. In teaching, teachers should pay attention to fully explore the dual role of students' left brain and right brain thinking, so that they can play a vital role in mathematics teaching, help students improve their logical thinking ability, improve the comprehensive ability of mathematics, and can more adapt to the needs of the society and the development of The Times.

III. ABSTRACT THINKING IN MATHEMATICS EDUCATION

The general law of abstract thinking roughly includes two aspects: one is how abstract thinking is carried out in mathematical research; Second, how to develop abstract thinking in mathematics education.

A. The research stage of mathematical abstract thinking

The research on mathematical abstract thinking can be roughly divided into the following four stages.

(1) The first is the study of mathematical abstraction, that is, the study of mathematical representation. Mathematical abstraction generally starts from the representations that mathematical cognition initially contacts, which does not mean that all representations can become the research objects of mathematical abstraction. People will have a deep discussion on some phenomena with high frequency in mathematical research and application, which indicate some regularity, and carry out consciously abstract thinking activities. Most of the initial mathematical representations were generated in production activities, such as the representation of geometric figures obtained in land survey, pottery making and other practical activities, figures from trade and timekeeping activities. In general, mathematical representations can only capture a few particular phenotypes, and the task of the mathematical worker is to discover the general from the particular, as m. Klein did, that there is an art of discerning a unified idea in a disparate problem and of bringing together the necessary materials to illustrate that unified idea. Only then can mathematical representations become useful material.

(2) It mainly analyzes various specific mathematical attributes, analyzes related attributes, removes non-essential attributes, and only retains the quantitative relations that can reflect essential attributes. For some newly discovered quantitative relations, it is often necessary to have a new symbol to represent, in essence, this is an innovative process. Isomorphism refers to that mathematical problems with the same quantitative relationship are the same in structure. Only on the basis of difference can isomorphism abstract the common essential attributes or characteristics of the same kind of mathematical problems.

(3) For the already known mathematical facts, combined with relevant existing mathematical theories, we can further determine their essential attributes and characteristics. New mathematical concepts must be formed and developed on the basis of the original mathematical system, and the connection of new and old knowledge requires strict logical reasoning. As Hilbert said, a new problem, especially when it comes from the world of external experience, must be transplanted to the branches of established mathematical achievements according to strict rules before it can blossom and bear fruit. Of course, the formation of a mathematical concept is very difficult, because the definition reflects not only the operation rules themselves, but also the internal relations between concepts, which can only be determined when mathematics has developed to a certain extent.

(4) Once a mathematical concept is basically established, it needs a long and repeated refining process. The connotation of the concept needs to be refined and deepened continuously; The extension of concepts also needs to be constantly expanded. For example, the multiplication operation is between the numbers at the beginning, with the continuous deepening and expansion of mathematics, slowly expanded to polynomial, matrix and other aspects of the multiplication operation. Although their product rules are different, there are some similarities. For example, multiplication in set theory, as an algebraic operation, is a mapping of the product set of two sets to another set, which further develops and refines the connotation and extension of multiplication.

In short, abstract thinking is based on the basic, a large number of individual cases, in observation, analysis, induction, summary of the law, to find the essence. The generation of mathematical concepts and the acquisition of mathematical methods usually conform to this law.

B. The realization of abstract thinking in mathematics teaching

In mathematics teaching, for teachers and students, the

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focus of their study and research on mathematics is not how abstract thinking is carried out. The teaching purpose of teachers is to complete the teaching requirements of mathematics courses and help students to learn mathematics better. Students learn mathematics in order to improve their mathematical performance, and through learning how to apply abstract thinking to mathematics to help improve their mathematical ability, therefore, for teachers, how to realize and develop the key abstract thinking ability in mathematics education.

In mathematics teaching, teachers often start from some typical and concrete problems when describing abstract concepts, which accords with the historical process of the occurrence of mathematical concepts. If teachers want students to understand and master abstract mathematical concepts, they should use some typical examples to help teaching. Simply memorizing abstract concepts will make students feel that mathematical concepts are empty and have no substance. To make students fully understand abstract concepts, it is necessary to illustrate them with typical examples.

Such as the concept of the definite integral, uses the "direct generation of song" and "close to" mathematics thought, the problem into four steps: segmentation, replacing, summation approximation and limit of four steps, through the study of the division of graphics, in every small local, adopt "with the method of straight and curve", narrow the area of the rectangle to approximate instead of narrow the area of the curved trapezoid (as shown in figure.1), In order to effectively reduce the error, the segmentation is gradually refined (as shown in Figure.2). The finer the segmentation is, the smaller the error will be. Although curves and straight lines are fundamentally different, as described by dialectics of nature, quantitative change and qualitative change are both different and related, and when equivalent change reaches a certain level, qualitative change will inevitably result. Finally, the exact value of the problem is obtained by taking the limit.

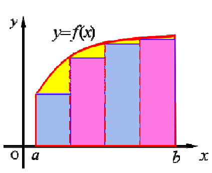

Fig. 1. Area partition diagram of a curved trapezoid

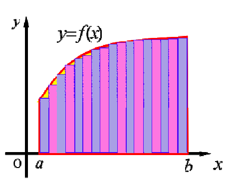

Fig. 2. Area subdivision of a curved trapezoid

Example is the basis for people to understand and use abstract concepts. Of course, this basis has limitations. In order to truly understand and use abstract concepts, we must get rid of the limitations of the example itself. For example, when students draw right triangles, most of the time they draw the right triangle at the bottom, and very rarely at the top or in other directions. This is because the right triangle below the right Angle first comes into the mind of students. When students learn and understand the right triangle, the first contact is such a triangle. As a result, the practice of seeing right-angled triangles in other directions requires overcoming the previous interference, a psychological phenomenon known as "negative transfer". After students practice right triangles in different directions, it will be easier for them to transform concepts. However, when they meet more complex situations, it will be difficult to eliminate the negative transfer effects brought by examples. This is also a major reason why many students feel at a loss when they see the same topic presented in another form in study.

In order to better learn mathematical concepts, students should learn and understand mathematical concepts according to examples, but also pay attention to overcome the negative influence brought by examples. So how can students overcome such negative distractions? First of all, it is necessary to have a full understanding of the connotation and denotation of the abstract concept, grasp the essence of the concept, and have a certain understanding of its different application and proof in different instances; Then, to examine the object from the original problem in the complex connection separated, directly to the concept of the definition of a separate research and analysis, learn to "change a point of view".

With the continuous development of mathematical system, mathematical concepts and definitions will also change, which conforms to the objective law of mathematical development. For concepts with similar or similar names in different systems, pay attention to their differences so that you can better understand and grasp the abstract concepts. For example, ordinary addition and Abelian group addition, ordinary geometry and linear space, etc., all have different meanings. With the change of mathematical system, understanding the change of corresponding mathematical concept can better understand mathematical concept.

To sum up, the limitation of the development of abstract thinking ability in mathematics education mainly comes from the aspect that is concrete and not limited by concrete. Mathematics teachers should according to the mathematical theory system of abstraction and structure, to help students construct the thinking foundation of abstract thinking, make students realize the conversion of abstract and concrete, consciously help them achieve the development of mathematics abstract thinking from junior to senior, achieve the purpose of training students in abstract thinking ability, improve the students' mathematical ability, let the students get high marks in mathematics at the same time, Also enjoy the fun of math learning.

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IV. Analysis on the Existing Problems and Improvement Measures of Students' Mathematical Abstract Thinking

A. Problems existing in students' mathematical abstract thinking

Although some students spend a lot of time on math study every day, they still can't improve their math scores, which is mainly due to their low level of mathematical abstract thinking ability. Specific performance in the following aspects.

(1) In the process of mathematical knowledge learning slow start, slow response. Students with better math scores and easier math learning will quickly enter the learning state in math class and master the key points of math quickly. Mathematics learning is difficult for students, it is always difficult to quickly enter the learning state, the acceptance of new knowledge is slower. In the whole middle school mathematics learning process, compared with other subjects, the amount of mathematics knowledge is larger, but the learning time is limited, the teacher's teaching task is to impart knowledge to students within the limited class hours. Students who enter the learning state slowly and react slowly often feel that the teacher has already started the explanation of the next knowledge point before they have mastered the previous knowledge point. Everyone is in the same math class, and the teacher doesn't stop and wait because a few students can't get into learning mode. Therefore, accumulated over a long period of time, the state of the poor students in mathematics learning, slowly interrupted their exploration of mathematical knowledge, can only mechanically carry on the record of knowledge points.

(2) Students' abstract thinking ability starts from a low point, resulting in poor learning results. Students need to step back from abstract thinking. Retreat is to retreat to the most specific and even the most primitive place, even some of the low degree of abstraction of mathematical objects, also need students to have a realistic prototype as a tool for understanding. If they don't have a deep impression of it, and they don't practice it many times, they won't be able to master it. In other words, if the starting point of students' abstract ability is too low, their learning efficiency will be greatly reduced, and the cultivation and development of abstract thinking ability will also be limited to a certain extent.

(3) Students' abstract thinking span is smaller. Students' abstract thinking level is developed step by step, many students' abstract thinking ability is relatively low, if the teacher's teaching content has a little jump or disjointed. It's hard for them to understand. If you don't dig deeper, the phenomenon of swallowing dates will appear. If students do not solve one or two abstract difficult problems in time and make in-depth analysis, they may feel at a loss what to do and confused in the process of learning mathematics. The experiential abstract thinking has an influence on the further improvement of students' mathematical thinking. The primary task of cultivating students' abstract thinking ability in mathematics teaching is to realize the reasonable connection of knowledge before and after, and promote the transition from image thinking to abstract thinking better.

B. Teaching measures to improve students' abstract thinking ability

(1) Abstract concepts should be visualized in mathematics teaching to help students better understand. For example, from junior high school, algebra part of learning began to involve geometric concepts. In geometry knowledge structure, the first concept is the plane, such as classroom, table, blackboard, etc. Teachers can use the concrete things around, visualize the abstract mathematical concept, let students observe, and observe the physical object and the abstract mathematical concept of the association, after the abstract concept of the abstract concept, and then use the concrete object for abstract generalization. For example, for the linear plane relationship, students can use the classroom podium or walls to help understand. Turning abstract mathematical models into concrete things directly existing in real life can help students narrow the span of thinking and make students better understand these abstract mathematical knowledge.

(2) Abstract conclusion should be concretized in mathematics teaching. If students do not really dig out the exact meaning of the given question, they cannot solve it and their thinking will be limited. Therefore, students should carefully and repeatedly examine all the information of known conditions in the problem and find the exact meaning of these conditions and data when they are practicing mathematical exercises. To do this, students need to enhance the flexibility of thinking, and in learning need not be afraid of difficulties, more use of the brain, can not be afraid of the difficulties encountered in training psychology. Teachers in this kind of thinking training, we should pay attention to encourage and guide students to overcome the psychological barriers of fear of difficulties, have interest in mathematics learning, increase their learning confidence, improve the efficiency of mathematics learning.

(3) Abstract methods should be popularized in mathematics teaching. Like the proof of mathematical induction. Can use simple life examples, such as a string of firecrackers connected to each other lead, let the students experience the role of this lead, if the lead is not connected, what will happen? Let students more clearly know, understand the importance of mathematical induction verification, make them understand induction hypothesis is essential in mathematics learning. This learning method, starting from familiar problems to guide students to reflect, can make students' thinking more active, can make students unconsciously complete the transition from image thinking to abstract thinking. Students can check the correctness of inductive results and prove the correctness of the process through checking calculation, so as to enhance the reflection of thinking.

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(4) We should pay attention to the organic infiltration of dull teaching in mathematics teaching. In the process of mathematics teaching, teachers can infiltrate plain teaching into classroom teaching. The learning difficulty of plain teaching is relatively small, the degree of abstraction of mathematical research object is relatively low, and the basic knowledge is relatively simple.

In a word, the most important point to improve students' mathematical abstract thinking ability lies in the guidance of teachers and their own training in daily learning. Through a lot of training, can improve the ability of abstract thinking. For the students with low level of abstract thinking ability, teachers should be patient, help them to formulate feasibility plans step by step, establish their confidence in mathematical abstract thinking training, improve the interest in mathematical learning, and achieve the teaching purpose of cultivating and improving abstract thinking ability. For students, teachers choose appropriate difficulty of mathematics learning, students can better focus on training thinking.

V. CONCLUSION

Mathematical abstract thinking ability is an important part of mathematical thinking ability. According to the four stages of mathematical abstract thinking research, this paper probes into the realization of abstract thinking in mathematics teaching. In view of the characteristics of mathematical abstract thinking, combined with the problems existing in the abstract thinking of middle school students in mathematics learning, the concrete teaching measures to improve students' abstract thinking ability are given.

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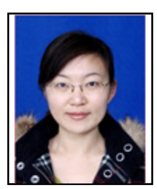

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