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Application of Block Matrix in Determinant and Inverse Matrix

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***Abstract*—On the basis of introducing the basic concepts of block matrix, combining with the related properties of block matrix, this paper analyzes the application of block matrix in the inverse matrix of high-order determinant and high-order matrix through concrete examples.**

***Index Terms*—Block matrix; Determinant calculation; inverse matrix.**

I. Introduction

Block matrix and determinant are important parts of linear algebra, and are important research tools. Block matrix can provide the idea of "packing" for the inversion of high-order matrix and the calculation of high-order determinant, which is helpful to simplify the calculation [1]. This paper first analyzes the theory of block matrix in the inversion of high-order determinant and special high-order matrix, and discusses the application of block matrix in theoretical operation and engineering technology with specific examples [2].

The definition of block matrix divides the rows and columns of a matrix into several groups, thus the matrix is divided into several sub-matrices, and the matrix is regarded as composed of these sub-matrices, which is called block of matrix. This matrix composed of submatrices is called block matrix [3].

The operation of block matrix is the same as that of digital matrix in form, and block matrix can be added, multiplied and multiplied as digital matrix [4]. As long as the operation matrix is partitioned properly, it will bring a lot of convenience to the calculation of matrix operation and determinant. There are many partitioning methods of matrix, but what kind of partitioning method should be adopted, it is necessary to select the best and simplest according to the problems encountered.

II. Application of Block Matrix in Determinant Calculation

When calculating the value of special determinant, we can use the idea of matrix partitioning to make the determinant structure to be calculated simpler, thus playing the role of simplifying the calculation [2]. The specific calculation theorem is as follows:

A. *Theorem 1*

Let A and B be matrices of order $m\times n$ and $n\times m$ respectively, then $\begin{vmatrix} E_m & B \\ A & E_n \end{vmatrix} = |E_m - AB| = |E_n - BA|$.

Example1 Calculate the determinant:

$$D=\begin{vmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \\ 4 & -6 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{vmatrix}.$$

Method 1. The determinant was transformed into an upper triangle.

$$D=\begin{vmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & -6 & 9 & -12 \\ 0 & 3 & 4 & -5 \end{vmatrix}=\begin{vmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 27 & 18 \\ 0 & 0 & -5 & -20 \end{vmatrix}$$

$$=45\begin{vmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & -1 & -4 \end{vmatrix}=45\begin{vmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 3 & 2 \end{vmatrix}$$

$$=45\begin{vmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -10 \end{vmatrix}=-450.$$

Method 2. According to Theorem 1,

$$D=\begin{vmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 5 \\ 4 & -6 & 1 & 0 \\ 2 & 3 & 0 & 1 \end{vmatrix}=\left|\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}-\begin{pmatrix} 4 & -6 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 3 & 5 \end{pmatrix}\right|$$

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$$= \left|\begin{pmatrix}1&0\\0&1\end{pmatrix} - \begin{pmatrix}-26&-18\\5&21\end{pmatrix}\right| = \begin{vmatrix}27&18\\-5&-20\end{vmatrix}$$

$$= 45\begin{vmatrix}3&2\\-1&-4\end{vmatrix} = -450.$$

B. *Theorem 2*

Let $\begin{pmatrix}A&B\\C&D\end{pmatrix}$ be a square matrix and $|A| \neq 0$, then

$$\begin{vmatrix}A&B\\C&D\end{vmatrix} = |A|\left|CA^{-1}B+D\right| \text{ [5].}$$

In fact, because

$$\begin{pmatrix}E&0\\CA^{-1}&E\end{pmatrix}\begin{pmatrix}A&B\\C&D\end{pmatrix} = \begin{pmatrix}A&B\\0&CA^{-1}B+D\end{pmatrix},$$

so

$$\begin{vmatrix}A&B\\C&D\end{vmatrix} = \begin{vmatrix}E&0\\CA^{-1}&E\end{vmatrix}\begin{vmatrix}A&B\\C&D\end{vmatrix}$$

$$= \begin{vmatrix}A&B\\0&CA^{-1}B+D\end{vmatrix} = |A|\left|CA^{-1}B+D\right|.$$

Example2. Calculate the determinant:

$$D = \begin{vmatrix}2&1&-1&4\\5&3&2&1\\3&-1&-2&3\\2&-1&3&1\end{vmatrix}.$$

According to Theorem 2,

$$D = \begin{vmatrix}2&1&-1&4\\5&3&2&1\\3&1&-2&3\\2&-1&3&1\end{vmatrix}$$

$$= \begin{vmatrix}2&1\\5&3\end{vmatrix}\left|\begin{pmatrix}3&1\\2&-1\end{pmatrix}\begin{pmatrix}2&1\\5&3\end{pmatrix}^{-1}\begin{pmatrix}-1&4\\2&1\end{pmatrix} + \begin{pmatrix}-2&3\\3&1\end{pmatrix}\right|$$

$$= \left|\begin{pmatrix}3&1\\2&-1\end{pmatrix}\begin{pmatrix}3&-1\\-5&2\end{pmatrix}\begin{pmatrix}-1&4\\2&1\end{pmatrix} + \begin{pmatrix}-2&3\\3&1\end{pmatrix}\right| = -40.$$

III. Application of Block Matrix in Matrix Inversion Operation.

A. *Theorem 3*

Let $A = \begin{pmatrix}A_1&&&\\&A_2&&\\&&\text{O}&\\&&&A_n\end{pmatrix}$ be a block diagonal matrix and $A_i\ (i=1,2,\cdots,n)$ are invertible matrices, then

$$A^{-1} = \begin{pmatrix}A_1^{-1}&&&\\&A_2^{-1}&&\\&&\text{O}&\\&&&A_n^{-1}\end{pmatrix}.$$

Example3. Find the inverse of this matrix:

$$Q = \begin{pmatrix}2&-3&0&0&0\\-1&2&0&0&0\\0&0&-1&2&3\\0&0&4&-5&6\\0&0&7&8&-9\end{pmatrix}.$$

Let

$$A = \begin{pmatrix}2&-3\\-1&2\end{pmatrix}, B = \begin{pmatrix}1&2&3\\2&3&1\\3&1&2\end{pmatrix},$$

then

$$A^{-1} = \begin{pmatrix}2&3\\1&2\end{pmatrix}, B^{-1} = \frac{1}{18}\begin{pmatrix}-5&1&7\\1&7&-5\\7&-5&1\end{pmatrix}$$

can be solved.

According to Theorem 3,

$$Q = \begin{pmatrix}A^{-1}&0\\0&B^{-1}\end{pmatrix} = \begin{pmatrix}2&3&0&0&0\\1&2&0&0&0\\0&0&-\frac{5}{18}&\frac{1}{18}&\frac{7}{18}\\0&0&\frac{1}{18}&\frac{7}{18}&-\frac{5}{18}\\0&0&\frac{7}{18}&-\frac{5}{18}&\frac{1}{18}\end{pmatrix}.$$

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B. *Theorem 4*

In the block matrix $G=\begin{pmatrix} A & 0 \\ C & B \end{pmatrix}$, A and B are invertible matrices, then $G^{-1}=\begin{pmatrix} A^{-1} & 0 \\ -B^{-1}CA^{-1} & B^{-1} \end{pmatrix}$ [3].

Example4. Find the inverse of this matrix:

$$G=\begin{pmatrix} 3 & -2 & 0 & 0 \\ -7 & 5 & 0 & 0 \\ -1 & -3 & 2 & 3 \\ 2 & 7 & 5 & 8 \end{pmatrix}.$$

Let

$$A=\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}, B=\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}, C=\begin{pmatrix} -1 & -3 \\ 2 & 7 \end{pmatrix},$$

then

$$A^{-1}=\begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}, B^{-1}=\begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix} \text{ can be solved.}$$

According to Theorem 4,

$$G^{-1}=\begin{pmatrix} A^{-1} & 0 \\ -B^{-1}CA^{-1} & B^{-1} \end{pmatrix}$$
$$=\begin{pmatrix} 5 & 2 & 8 & -3 \\ 7 & 3 & -5 & 2 \\ 385 & 163 & 0 & 0 \\ -248 & -105 & 0 & 0 \end{pmatrix}.$$

IV. Conclusion

From the above analysis, it can be seen that the main idea of block matrix is "packing", which is used to reduce the order and get closer to the special matrix. Block matrix plays an important role in the research of high-order determinant and high-order matrix. On the one hand, it can simplify the operation, on the other hand, it can also help us see clearly the structural characteristics of sub-blocks in the operation process [5].

With the help of block matrix, a large matrix operation can be transformed into several small matrix operations, which makes the internal relations of the matrix clearer and simplifies the calculation [6]. This paper mainly explains the application of block matrix method in determinant related calculation, shows the important role of block matrix in higher algebra, fully highlights the flexibility and high efficiency of block matrix in matrix calculation process, and reflects the extremely high practical value of this tool, so it is not only a problem-solving idea, but also a problem-solving path [7].

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