

# Crack Detection in Brake Disc by Modal Analysis

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**Abstract**— The aim of the thesis is to investigate techniques and parameters that could be used to identify crack if it exists in brake disc. Health monitoring for disc brake due to crack using crack detection techniques will minimize or reduce the failure that probably to occur. Crack changes the dynamic behaviour of the structure and by examining this change, crack size and position can be identified. Among few methods of detecting crack components and due to its feasibility of detection of fatigue crack, vibration –based crack detecting techniques through modal analysis are applied in this thesis. This method is based on the fact that change of physical properties(stiffness, mass and damping) due to crack that will manifest themselves as changes in component modal parameters. Experimental Modal Analysis (EMA) was performed on cracked disc and a healthy disc. The first three natural frequencies were considered as basic criterion for crack detection. To locate the crack, 3D graphs of the normalized frequency in terms of the crack depth and location are plotted. The intersection of these three contours gives crack location and crack depth.

**Index Terms**—About four key words or phrases in alphabetical order, separated by commas.

## I. INTRODUCTION

Damage identification methods are mainly based upon the shifts in natural frequencies or changes in mode shapes. NDT methods are often employed for detection of cracks in machine and structural components. All of these NDT techniques require that the location of the damage is known a priori and that the portion of the structure being inspected is readily accessible. In order to detect a crack by this method, the whole component requires scanning. The drawbacks of traditional localized NDT methods have motivated development of global vibration based damage detection methods.

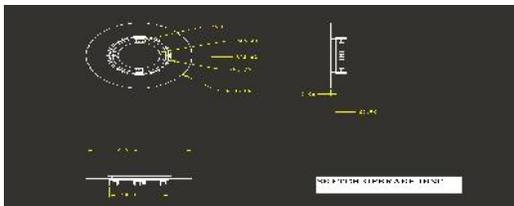


Fig 1. A thick annular disk with a hat structure simulates the brake rotor. Disk is clamped at the inner bolts and free at outer edge

**Table 1.** Geometric dimensions and material properties of the brake rotor.

Outer radius (a)	=216 mm
Inner radius (b)	=110 mm
Radii ratio ( = b/a)	= 0.509
Disk thickness (h)	=10 mm
Hat height (H)	= 23 mm
Density ( d)	=6900 Kg/m <sup>3</sup>
Young's modulus (E)	=110GPa
Poisson's ratio ( )	= 0.211



Fig.2. Brake Disc in assembly

## II. PROCEDURE FOR PAPER SUBMISSION

### 1. Vibration of Circular Disc Without Damping : An Analytical Approach

The equation of motion of a circular Kirchhoff plates in the polar coordinate system reads

$$\rho h w_{,tt} \mid D \nabla^4 w = 0, \quad (1)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}.$$

Substituting a solution of the form

$$w(r, \phi, t) = W(r, \phi) e^{i\omega t} \quad (2)$$

gives

$$\nabla^4 W - \gamma^4 W = 0,$$

$$\text{or } (\nabla^2 + \gamma^2)(\nabla^2 - \gamma^2)W = 0, \tag{3}$$

where

$$\gamma^4 = \frac{\rho h \omega^2}{D}. \tag{4}$$

For a plate with suitable boundary conditions one may assume a separable periodic solution

$$W(r, \phi) = R(r)e^{im\phi}. \tag{5}$$

Using this in (3) yields

$$\left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} + \left( \gamma^2 - \frac{m^2}{r^2} \right) \right] \left[ \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \left( \gamma^2 + \frac{m^2}{r^2} \right) \right] R_m = 0. \tag{6}$$

One can find solutions of  $R_m(r)$  as

$$R_m(r) = A_m(r) + B_m(r), \tag{7}$$

$$\frac{d^2 A_m}{dr^2} + \frac{1}{r} \frac{dA_m}{dr} + \left( \gamma^2 - \frac{m^2}{r^2} \right) A_m = 0, \tag{8}$$

$$\text{and } \frac{d^2 B_m}{dr^2} + \frac{1}{r} \frac{dB_m}{dr} - \left( \gamma^2 + \frac{m^2}{r^2} \right) B_m = 0. \tag{9}$$

Where

The Bessel differential equation (8) has the general solution

$$A_m(r) = C_1 J_m(\gamma r) + C_2 Y_m(\gamma r), \tag{10}$$

Where

$C_1$  and  $C_2$  are arbitrary constants, and  $J_m(.)$  and  $Y_m(.)$  are, respectively, the Bessel functions of first and second kinds of order  $m$ . For the modified Bessel differential equation (9), the solution can be written as

$$B_m(r) = C_3 I_m(\gamma r) + C_4 K_m(\gamma r), \tag{11}$$

Where

$C_3$  and  $C_4$  are arbitrary constants, and  $I_m(.)$  and  $K_m(.)$  are known as, respectively, the modified Bessel functions of first and second kinds of order  $m$ . Therefore, the solution of  $R_m(r)$  is given by

$$R_m(r) = C_1 J_m(\gamma r) + C_2 Y_m(\gamma r) + C_3 I_m(\gamma r) + C_4 K_m(\gamma r). \tag{12}$$

The value of  $\gamma$ , and the constants in (12) are determined from the boundary conditions.

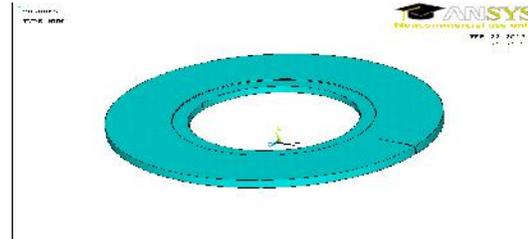


Fig.3. Equivalent Disc Model of Cracked Brake Rotor

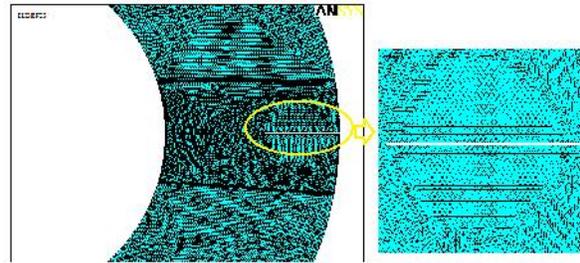


Fig.4. Meshed Model of Rotor with crack

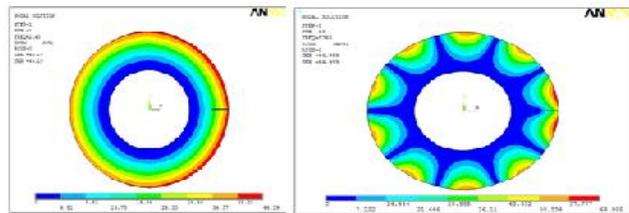


Fig. 2.3(a)

Fig. 2.3(b)

Fig..2.3(a). 1<sup>ST</sup> Mode shape of cracked rotor at 2143 Hz

Fig.2.3(b). 10<sup>th</sup> mode shape of 5Nodal Diameters at 5788 Hz

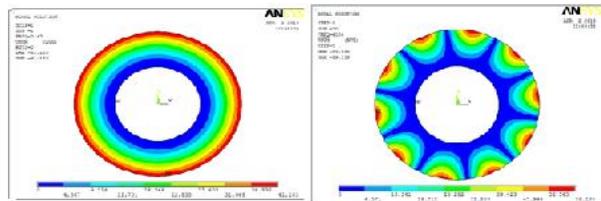
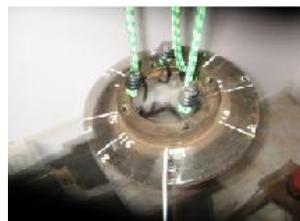
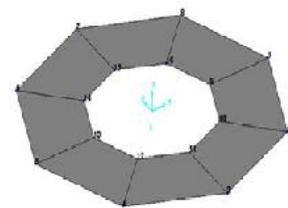


Fig. 7. 1<sup>st</sup> Mode shape of uncracked disc at 2147 Hz

Fig.8. 10<sup>th</sup> mode shape of 5ND of uncracked disc at 6004 Hz



[1] Fig.9. Brake Rotor with accelerometer



[2] Fig.10. Test Node points on the structure

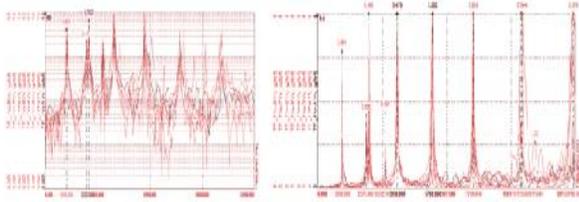


Fig.11.FRF Plot in Y-axis

Fig.12. Disc Modal Peaks

**Table.2. Comparison between Experimental and Numerical Results**

[3] No	[4] Numerical (Hz)	[5] Experimental (Hz)	[6] % Deviation
[7] 1	[8] 1682.9	[9] 1200	[10] 40.24
[11] 2	[12] 1699.6	[13] 2375	[14] 39.73
[15] 3	[16] 2759.6	[17] 2525	[18] 9.29
[19] 4	[20] 3268.9	[21] 3325	[22] 1.716

**Change in the Natural Frequencies of Cracked and Uncracked Brake rotor Disc (FEA Results)**

Mode No	Uncracked Disc		Cracked Disc		%Decrease in Nat Freq (Hz)
	Mode No	Natural Frequency(Hz)	Mode No	Natural Frequency(Hz)	
1	2148.9	2148.9	1	2143.2	0.172
2	2210.8	2210.8	2	2202.7	0.368
3	2210.8	2210.8	3	2208.2	0.118
4	2490.9	2490.9	4	2489.0	1.206
5	2490.9	2490.9	5	2478.3	0.508
6	3186.9	3186.9	6	3098.1	2.235
7	3186.9	3186.9	7	3151.3	0.492
8	4339.9	4339.9	8	4200.2	2.08
9	4339.9	4339.9	9	4281.5	1.345
10	6004.3	6004.3	10	5780.1	3.957

**Table.3.Natural frequency result from EMA at free-free BC of disc**

S.No	Nat freq(Hz)	TF in g/Hz	Damping coefficient
1	1200	3.11	.011701
2	2375	2.093	.0089
3	2525	6.147	.004514
4	3325	2.162	.006078
5	3950	5.479	.00542
6	5700	5.262	.0029836

[1] J.He , and Z.F.Fu, Modal Analysis, Oxford, Butterworth-Heinemann,2001.  
 [2] M.Z.Hassan, Brake disc performance: FEA,Msc.Thesis,Coventry University,Coventry,UK,2004.  
 [3] DJ Ewins: Modal Testing: Theory, Practice and Application, Research Studies Press Ltd., New York(1984)  
 [4] A.L.Gyenkenyesi,J.T.Sawichki and G.Y.Baaklini, "Vibration Based Crack Detection in a Rotating Disk" Part I- Analytical Study,NASA,2003.  
 [5] HaileleoulSahle, Cracked Detection in Rotating Disc, MSc.Thesis, Addis Ababa University,2004.  
 [6] J.C.Bae and J.A.Wickert 2000 Journal of Sound and Vibration 235(1),117-132.Free Vibration of Coupled Disk-Hat Structures.  
 [7] Free vibration of coupled disk-hat structures—J.C.Bae and J.A.Wickert  
 [8] Hwang and Kim --- Damage detection in structures using a few FR measurement.  
 [9] A study of Disc Brake high frequency squeals and Disc in plane/out of plane Modes—Michael Yang.  
 [10] Modal Analysis of Car Brake by Ahmad Zaki Bin CheZainolAriff.

III. REFERENCES

(Periodical style)