

# Comparison of Response of PID, FLC, MPC Controllers for Flow Tank System

V. Manimekalai<sup>1</sup>, B. Prashanth<sup>2</sup>, V. Sivashanmathi<sup>3</sup>, K. Surya<sup>4</sup>

Assistant Professor, Electronics and Instrumentation Department, Kumaraguru College of Technology, Coimbatore-49, India<sup>1</sup>

UG Student, Electronics and Instrumentation Department, Kumaraguru College of Technology, Coimbatore-49, India<sup>2</sup>

UG Student, Electronics and Instrumentation Department, Kumaraguru College of Technology, Coimbatore-49, India<sup>3</sup>

UG Student, Electronics and Instrumentation Department, Kumaraguru College of Technology, Coimbatore-49, India<sup>4</sup>

**Abstract** - In this paper, a brief explanation about different types of controllers and their tuning methods for a single flow tank system have been discussed. This paper mainly concern with modelling and designing of controllers for single tank system along with its simulation. Here theoretical and practical modelling for single tank system are identified and they are simulated with three different controllers such as PID (with controller tuning methods such as Relay Auto-Tuning, Ziegler-Nichols and Tyreus-Luyben), FLC and MPC to find the controller that provides maximum efficiency. They are designed using MatLab and compared their output responses for unit step response. Implementation of PID, FLC and MPC controllers is done by using the tool of MatLab-Simulink.

**Keywords** - Flow tank system simulation, Fuzzy logic controller, PID different tuning methods, Relay auto-tuning, Model Predictive Controller, Zeigler-Nichols, Tyreus-Luyben.

## I. INTRODUCTION

Flow control is a critical need for many industrial processes. Chemical industries constitute of a very vast, complex and sensitive processes. It is not that much easy to handle such a complex network of chemical processes. Every processes in a plant has operating conditions which are to be maintained. The violation of these operating conditions may be dangerous and may even cause human death. In these industries, the manipulating variables are mainly controlled by means of different controller algorithms. PI and PD<sup>[5]</sup> controller is one of the earlier control strategies which has more settling time and offsets on the output whereas Proportional-Integral-Derivative controller algorithm has a simple control structure which have been used for its robustness. But implementation of PID<sup>[3][4][5]</sup> control for a flow process leads to oscillatory response and large settling time. In order to overcome these issues different tuning methods for PID algorithm is introduced such as Ziegler-Nichols<sup>[3][4][7][5]</sup>, Tyreus-Luyben<sup>[4]</sup> and Relay-

Auto-Tuning<sup>[1][4][6][9]</sup> methods. Among those methods<sup>[10]</sup>, Relay Auto-tuning technique is designed efficiently to define these controller parameters. It has auto-tuning switch and by closing this switch PID parameters are computed and transferred to the controller automatically. Later Fuzzy Logic Controller algorithm<sup>[3][5]</sup> and Model Predictive Controller algorithm<sup>[2]</sup> was introduced for much faster response. Implementations of these controllers improve the response of the process to much higher level than previous algorithms. In this paper, we use to control the flow by means of FLC, MPC and PID (via three different tuning methods). Performance analysis of these controllers is done by the use of MATLAB and simulink. Comparison of various time domain parameters<sup>[11]</sup> is done to prove that the MPC has small overshoot and fast response than the FLC and PID algorithm.

## II. MODELING OF THE SYSTEM

### A. Mathematical Modeling of the System<sup>[8]</sup>

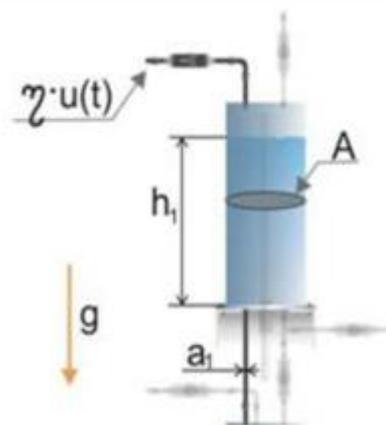


Fig. 2.1 Single Tank System

a) Taking Mass Balance:

$$\frac{dv}{dt} = A \frac{dh}{dt} = q(\text{in}) - q(\text{out})$$

(1)

Where  $v$  - volume of tank,  $q_{(in)}$  - inlet,  $q_{(out)}$  - outlet

b) Bernoulli's Law:

$$P + \frac{1}{2} \rho v^2 + \rho gh = \text{const} \quad (2)$$

Where  $P$  - Pressure,  $v$  - Velocity,  $h$  - Height,  $\rho$  - Density  
At surface, tank velocity  $v=0$ , bottom height,  $t=0$   
it gives

$$q_{(out)} = a\sqrt{2gh} \quad (3)$$

c) Pump flow:  $q_{(pump)} = \eta u$  (4)

By simple expansion in non linear form

$$\frac{dh_1(t)}{dt} = \eta u(t) - \frac{a_1}{A} \sqrt{2gh(t)} \quad (5)$$

Here  $a_1$  - tank 1 outlet,

$a_2$  - tank 2 outlet,

$A$  - Cross-sectional area of the tanks,

$g$  - Gravitational constant.

Heights of the tank is given as

$$h_{10} = \frac{1}{2} g \left( \frac{\eta u a_1}{A} \right)^2 \quad (6)$$

Making the above non-linear equation to linear form by linearization we get,

$$\sqrt{2gh_{10}(t)} = \frac{\eta u}{a_1}$$

On further solving, we get the above equation as

$$\frac{dh_1(t)}{dt} = -\frac{a}{2} \sqrt{2gh_{10}(t)} - \left( \frac{a_1}{A} \right)^2 \frac{g}{\eta u_0} (h_1(t) - h_{10}(t)) \quad (7)$$

At steady state with deviation variable can be given as

$$d\Delta h_1 = \eta \Delta u(t) - \left( \frac{a_1}{A} \right)^2 \frac{g}{\eta u_0} d\Delta h_1 \quad (8)$$

Taking Laplace transform on the equation 7, we get

$$\frac{\Delta H_1(s)}{\Delta u(s)} = \frac{\eta}{s + \left( \frac{a_1}{A} \right)^2 \frac{g}{\eta u_0}} \quad (9)$$

Hence the process transfer function for single tank system is given as

$$G_p(s) = \frac{\Delta H_1(s)}{\Delta u(s)} = \frac{K_p}{\tau s + 1} \quad (10)$$

Here  $K_p = \frac{\eta}{\left( \frac{a_1}{A} \right)^2 \frac{g}{\eta u_0}}$ ,  $\tau_p = \frac{1}{\left( \frac{a_1}{A} \right)^2 \frac{g}{\eta u_0}}$

The constant values of the system are given below.

- $\eta = 2.4 \times 10^{-3}$ ,
- $a_1 = 50.265 \times 10^{-6} \text{m}^2$ ,
- $A = 0.01389 \text{m}^2$ ,
- $g = 9.81 \frac{\text{m}^2}{\text{s}}$ ,
- $u_0 = 3.17v$ .

Substituting these values in equation 10, we get transfer function to as

$$G_p(s) = \frac{0.14218}{59.225s + 1}$$

B. To Find Transfer Function of the Plant in Real-time

The transfer function of the flow tank system is done by means of running the system in open loop configuration mode. To do so, the controller is set in the manual mode with parameters.

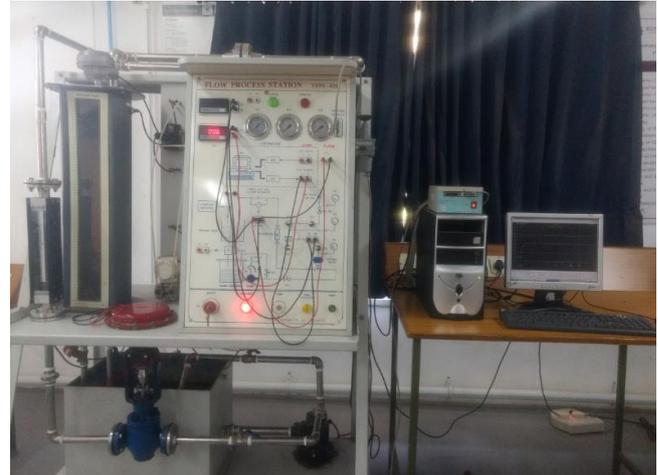


Fig. 2.2 Flow station

Table I. Parameters of Open Loop System

Mode	Setpoint	CV (%)
Manual	500	100

The output response of the open loop system is obtained as shown in the graph as fig.2.3

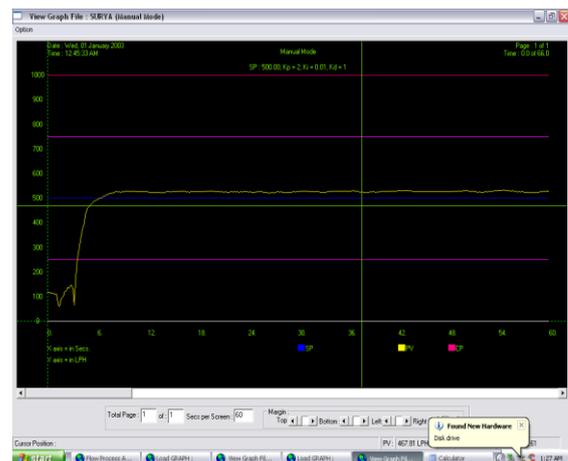


Fig. 2.3 Output Response of the Open Loop Flow Tank System

From the fig. 2.3, the values of  $K_p$  and  $\tau$  are obtained as shown in the table II.

Table II. Open Loop System Parameters

Mode	$K_p$	$\tau$
Manual	1.052	3.87

Substituting the values from the table 2.1 to the first order system transfer function

$$i.e. G_p(s) = \frac{K_p}{\tau s + 1}$$

We get the transfer function for the flow tank system as

$$G_p(s) = \frac{1.052}{3.87s + 1}$$

### III. CONTROLLER DESIGNS

#### A. PID Algorithm

A PID (Proportional-Integral-Derivative) Controller is being designed for a higher order system. It acts upon the derivative of the error, so it is most active when the error is changing rapidly. It serves to reduce process oscillations. But the response of this technique is not fast and reliable. Hence different tuning methods are being introduced for optimum control.

#### TUNING METHODS OF PID CONTROLLER

##### 1) Ziegler-Nichols Tuning Method:

Ziegler and Nichols closed-loop method is a tuning method that works as a trial and error method depending on sustained oscillations. This method is probably the most known and the most widely used method for tuning of PID controllers.

##### Tuning Procedure

1. Bring the process to the specified operating point of the control system to ensure that the controller is feeling representative process dynamic.
2. Turn the PID controller into a P controller by setting  $\tau_I=1$  and  $\tau_D=0$ . Initially set gain  $K_p=0$ .
3. Increase  $K_p$  until there are sustained oscillations in the signals in the control system. This  $K_p$  value is denoted the ultimate (or critical) gain,  $K_u$ .
4. Measure the ultimate (or critical) period  $P_u$  of the sustained oscillations.

With  $K_u$  and  $P_u$  found, Calculate the controller parameter values according to Table III.

Table III. Ziegler-Nichols Controller Parameter Settings

Controller Type	Proportional Gain, $K_p$	Integral Time, $T_I$	Derivative Time, $T_D$
P	$0.5K_u$	---	---
PI	$0.45K_u$	$P_u/1.2$	---
PID	$0.6K_u$	$P_u/2$	$P_u/8$

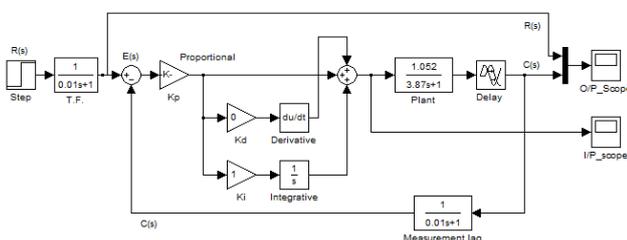


Fig. 3.1 Simulink Diagram of Ziegler-Nichols Method

The output signal with sustained oscillation for the unit step input is obtained as shown in the fig. 3.2.

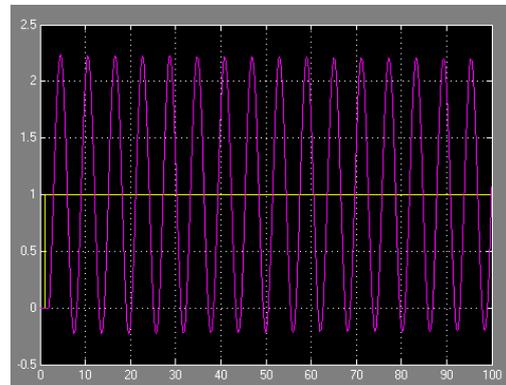


Fig. 3.2 Sustained Oscillation of the Plant

Table IV. Ziegler-Nichols Controller Parameters

$K_u$	$P_u$	$K_p$	$T_I$	$T_D$
2.83	7	1.70	3.5	0.88

##### 2) Tyreus-Luyben Tuning Method:

The Tyreus-Luyben procedure is quite similar to that of Ziegler-Nichols method but only difference is that the final control element settings. Also this method only proposes settings for PI and PID controllers. The values of Ultimate gain and period that are found in Z-N method are used in the table V to find the PID parameters. Like Z-N method this method is also time consuming and forces the system to margin if instability.

Table V. Tyreus-Luyben Controller Parameter Settings

Controller Type	Proportional Gain, $K_c$	Integral Time, $T_I$	Derivative Time, $T_D$
PI	$K_u/3.22$	$2.2P_u$	---
PID	$K_u/2.2$	$2.2P_u$	$P_u/6.3$

Table VI. Tyreus-Luyben Controller Parameters

$K_u$	$P_u$	$K_p$	$T_I$	$T_D$
2.83	7	1.30	15.4	1.11

##### 3) Relay Auto-Tuning Method:

A large industrial process may have hundreds of PID controllers that have to be tuned individually to match the process dynamics in order to provide good and robust control performance. The tuning procedure, if done manually, is very tedious and time consuming. Thus automatic tuning techniques draw more attention. By auto tuning, the controller is tuned automatically on demand from an operator or an external signal. Typically, the user will either push a button or send a command to the controller.

To determine the parameters of this method, the system is connected in a feedback loop as shown in fig. 3.3

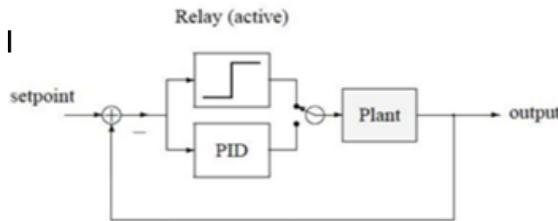


Fig. 3.3 Relay Auto-Tuning Block Diagram

The output signal of this method (as shown in fig. 3.4) is a periodic signal. The values of the ultimate gain  $K_u$  is given by the formula

$$K_u = \frac{4d}{\pi a}$$

Where  $d$  - relay amplitude and  $a$  - amplitude of the output. From the fig. 3.4, the values of  $a$  and  $d$  for the ultimate gain  $K_u$  is obtained. The ultimate period  $P_u$  is obtained by measuring the time elapse between two successive peaks of the output signal.

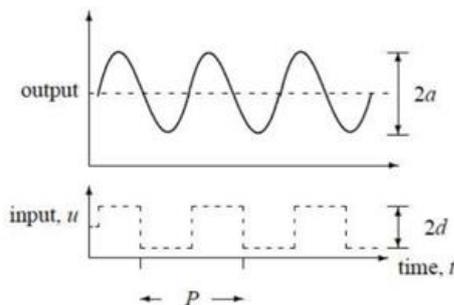


Fig. 3.4: Input & Output Signal of Relay Auto-Tuning

Here the values for the PID (ultimate gain  $K_u$  and ultimate period  $P_u$ ) are obtained according to the table VII.

Table VII. Relay Auto-Tuning Controller Parameter Setting

Controller type	Proportional gain, $K_p$	Integral time, $T_I$	Derivative time, $T_D$
P	$0.5K_u$	---	---
PI	$0.4K_u$	$1.25/P_u$	---
PID	$0.6K_u$	$1.25/P_u$	$0.12P_u$

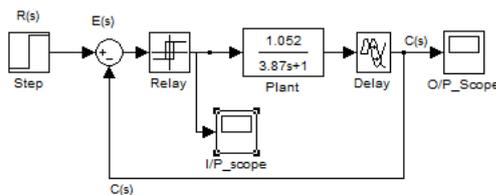


Fig. 3.5 Simulink Diagram of Relay Auto-Tuning Method

Table VIII. Relay Auto-Tuning Controller Parameters

$K_u$	$P_u$	$K_p$	$T_I$	$T_D$
4.055	3.5	2.43	1.8	0.44

### B. FLC ALGORITHM

Fuzzy Logic Controller is very simple conceptually. It consists of an input stage, a processing stage, and an output stage. The input stage has sensor or any other inputs, such as switches, buttons and relays. The processing stage invokes each appropriate rule and generates a result for each, then combines the results of the rules. Later, it is converted into a specific control output value. The most commonly preferred shape of membership functions is triangular, trapezoidal and gauss curves, but the shape is generally less important than the number of curves and their placement.

Here the processing stage is based on a collection of logic rules in the form of IF-THEN statements. Typical fuzzy control systems have dozens of rules.

Some advantages of FLC are as follows:

- It is robust in nature compared with other type of controllers.
- It works with very less precise inputs.
- It does not require any kind of fast processors.
- It uses only less data storage.

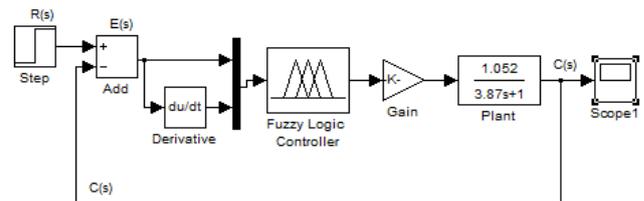


Fig. 3.6 Simulink Diagram of Fuzzy Logic Controller

### FLC Configuration:

In FIS editor, the number of inputs and outputs of the process are determined and also used to specify the rules for the operation as shown in fig. 3.9. The rules of the controller are added to the rule editor according to the table IX.

Table IX. Rules for Flow Tank System

u(t)	e(t)					
		NB	NS	ZO	PS	PB
$\Delta e(t)$	NB	NB	NB	NB	NS	ZO
	NS	NB	NB	NS	ZO	PS
	ZO	NB	NS	ZO	PS	PB
	PS	NS	ZO	PS	PB	PB
	PB	ZO	PS	PB	PB	PB

Here

- NB - Negative Big
- NS - Negative Small
- ZO - Zero
- PS - Positive Small
- PB - Positive Big

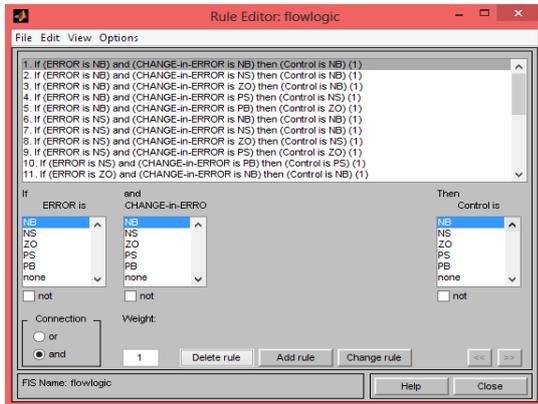


Fig. 3.9 Rules editor

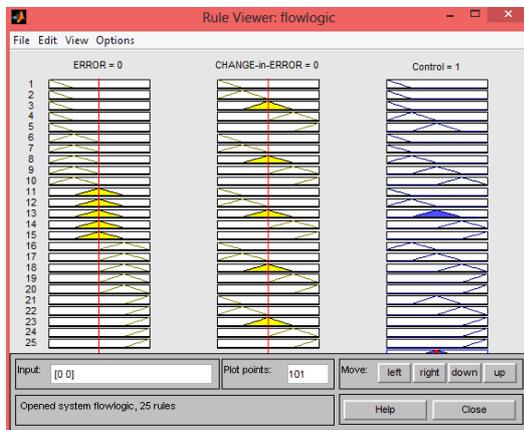


Fig. 3.10 Rule Viewer

### C. MPC ALGORITHM

Model Predictive Control (MPC) is widely applied in the process industries due to its capability to deal with constraints in an optimal fashion. MPC is based on predictions of set point tracking behaviour or disturbance rejection over both past controlled and manipulated variables measurements, in which each prediction is followed by an optimization routine to find the optimum input of the closed loop response.

It has three control variables that can be modified in order to get the optimized result from the controller such as Control interval, Prediction horizon and Control horizon.

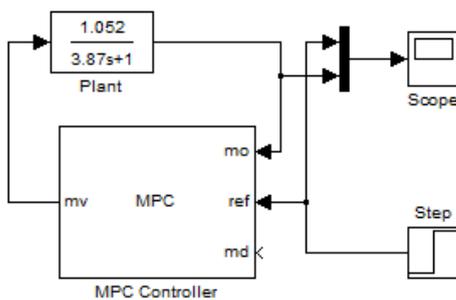


Fig. 3.11 Simulink Diagram of MPC

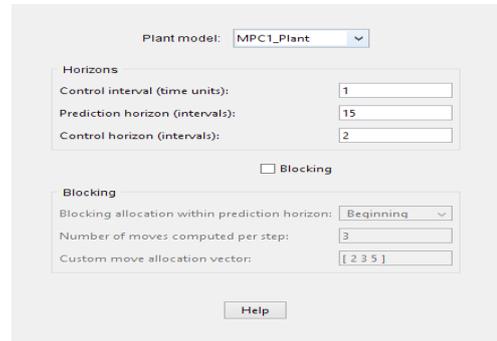


Fig. 3.12 Design Block of MPC

Table X. Controller Parameters of MPC Algorithm

Control interval	Prediction horizon	Control horizon
1	15	2

### IV. COMPARISON OF THE RESPONSE OF PID, MPC AND FLC ALGORITHM

Here the responses of the Proportional-Integral-Derivative controller (PID), Fuzzy Logic Controller (FLC) and Model Predictive Controller (MPC) are simulated as shown in fig.4.1 on a single unit and their time domain responses are compared to determine the best controller for the flow tank system.

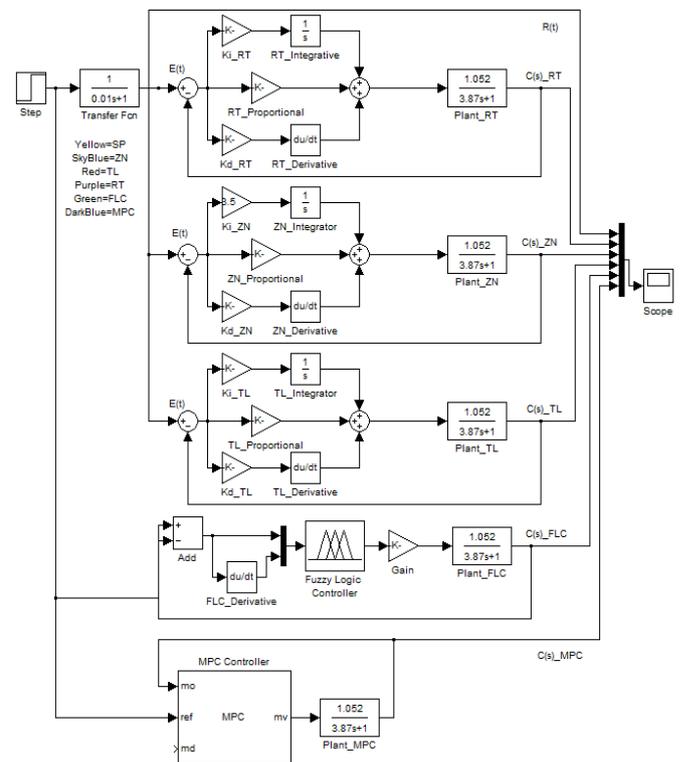


Fig. 4.1 Simulink Diagram of the Comparison Module



V. SIMULATION RESULTS

The output responses for PID Controller with different tuning methods for unit step input are shown below in fig. 5.1 to fig. 5.3.

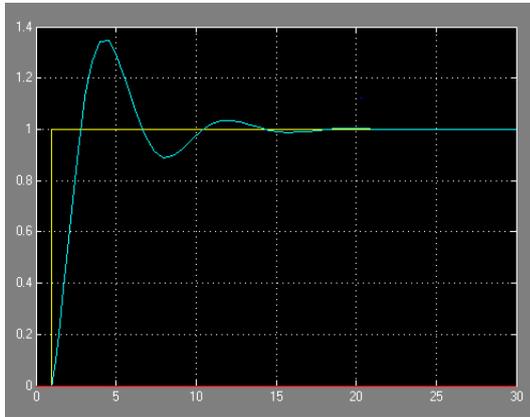


Fig. 5.1 Step Response of Ziegler-Nichols Method

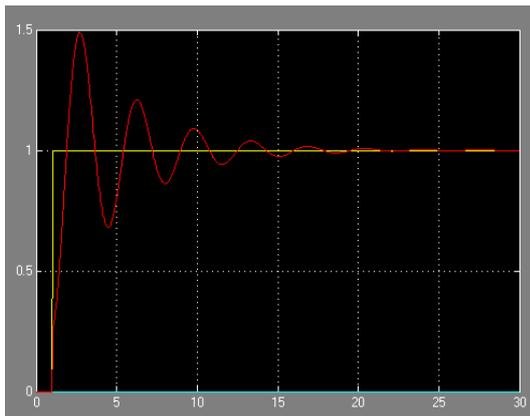


Fig. 5.2 Step Response of Tyreus-Luyben Method

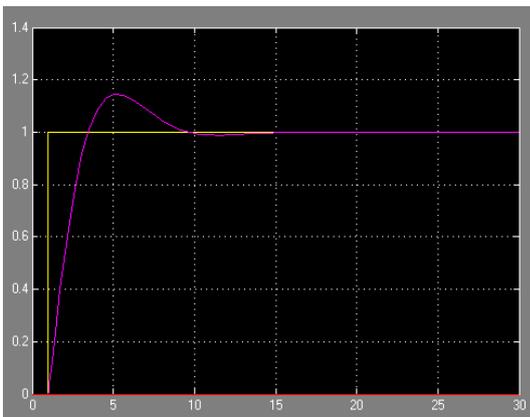


Fig. 5.3 Step Response of Relay Auto-Tuning Method

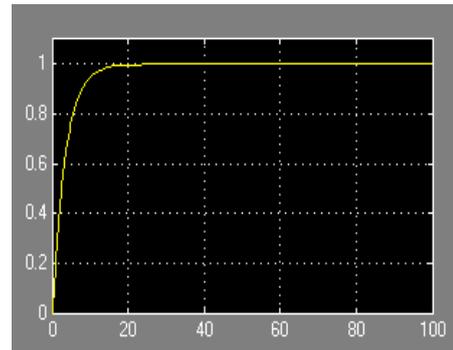


Fig. 5.4 Step Response of FLC Method

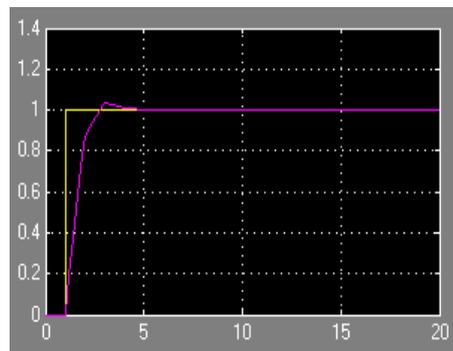


Fig. 5.5 Step Response of MPC Method

In order to compare the responses of the different controllers PID, FLC and MPC for unit step input of flow tank system, they are simulated together as shown in fig. 5.6. and the values of time domain responses are shown in table XI.

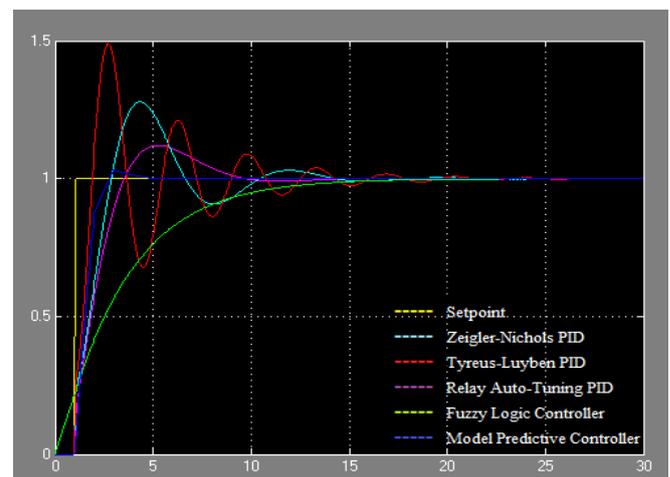


Fig. 5.6 Comparison of Responses of Different Controllers

Here the output responses for unit step input of flow tank system are shown below such as fig. 5.4 for FLC and fig. 5.5 for MPC.



Table XI. Comparison of Performances of Controllers

Parameters	Ziegler-Nichols PID	Tyres-Luyben PID	Relay Auto Tuning PID	FLC	MPC
Settling time	21	26	16	21	5
Over shoot	0.28	0.5	0.12	0	0.03

## VI. CONCLUSION

In this paper, the implementation of three controllers PID, FLC and MPC for single flow tank system had been done using MATLAB - Simulink. The responses of these controllers for unit step input are drawn with graphical output. Thus, by observing these responses on simulation, we can see that the Model Predictive Controller (MPC) has very less settling time with negligible peak overshoot. This proves that the MPC controller tends to provide higher stability and efficiency than any other controller techniques.

## VII. REFERENCES

- [1] Emine Dogru Bolat, "Real Time Temperature Control Of Oven Using Matlab-Simulink," in proceedings of the 11th WSEAS International Conference on SYSTEMS, Agios Nikolaos, Crete Island, Greece, 23-25 July 2007.
- [2] Applied Model Predictive Controller by Stefano Ciannella, University Of Toronto-Nov 2014.
- [3] Rahul Malhotra et al, "Boiler Flow Control Using PID and Fuzzy Logic Controller"- IJCSET, July 2011.
- [4] Sujit Anandrao Jagnade, Rohit Ashok Pandit, Arshad Ramesh Bagde, "Modelling, simulation and Control of flow tank system"-IJSR, 2013.
- [5] Smita Dinkar, "Design and Simulation of PD, PID and Fuzzy Logic Controller for Industrial Application"- IJICT, 2014.
- [6] K.J. Astrom, T. Hagglund, "Automatic tuning of simple controllers with specification on phase and amplitude margins"- Automatica 20 , 1984.
- [7] J. G. Ziegler and N. B. Nichols; "Optimum Settings for Automatic Controllers", Trans. ASME, Vol. 64, 1942, s.759-768.
- [8] Gatzke, E. P.; Meadows, E. S.; Wang, C.; Doyle III, F.J. "Model Based Control of a Four-tank System". Computer. Chem. Eng., 2000, 24, 1503-1509.
- [9] Johansson, K. H. "Relay Feedback and Multivariable Control", PhD Dissertation, Lund Institute of Technology, Lund, Sweden, 1997.
- [10] Mohammad Shahrokhi and Alireza Zomorodi, "Comparison of PID controller Tuning Methods", Feb, 2014.
- [11] S.Abirami, H.Kala, P.B.Nevetha, B.Pradeepa, R.Kiruthiga and P.Sujithra, "Performance Comparison of Different Controllers for Flow Process", International Journal of Computer Applications - 21, March 2014.