

## ADDITION AND SUBTRACTION USING LOGARITHMIC NUMBER SYSTEM FOR DIGITAL FILTERS

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### ABSTRACT

In this project presents techniques for low-power addition & subtraction in the Logarithmic Number System (LNS) and quantifies their impact on digital filter VLSI implementation. The impact of partitioning the LUT required for LNS addition & subtraction on complexity, performance, and power dissipation of the corresponding circuits is quantified. Two design parameters are exploited to minimize complexity, namely the LNS base and the organization of the LNS word. In addition, techniques for the low-power implementation of an LNS multiply-accumulate unit are investigated. The results are demonstrated by evaluating the power dissipation, complexity and performance of several FIR filter configurations comprising one, two or four MAC units.

**Keywords:** Logarithmic Number System, Computer Arithmetic, Digital Filter, FIR.

### I. INTRODUCTION

Data representation is an important parameter in the design of low-power processors since it affects both the switching

activity and hardware complexity. The Logarithmic Number System (LNS) has been investigated as an efficient way to represent data in special purpose VLSI processors, since it allows for simple arithmetic circuits under certain

conditions. In particular, LNS exploits the properties of the logarithm to reduce the basic arithmetic operations of multiplication, division, roots, and powers to binary addition, subtraction, and right and left shifts, respectively.

The dynamic range of floating point comes at the cost of lower precision and increased complexity over fixed point. Logarithmic number systems (LNS) provide a similar range and precision to floating point but may have some advantages in complexity over floating point for certain applications. This is because multiplication and division are simplified to fixed-point addition and subtraction, respectively, in LNS. However, floating point number systems have become a standard while LNS has only seen use in small niches.

### II. BASICS OF LOGARITHMIC NUMBER SYSTEM

The basic idea in LNS is to use logarithms to represent data. Since the logarithm of a negative number is not real, in order to represent signed numbers in LNS, the sign information is stored as a separate bit  $a_x$ , and used in combination with the logarithm of the magnitude of the number. Furthermore, since the logarithm of zero is not a finite number, an additional single-bit flag  $z_x$  is used to denote that a number is zero. Summarizing,  $X$  denotes the original number,  $x$  denotes the logarithm of the absolute value of  $|X|$ , and  $X_{LNS}$  is a triplet containing the sign bit, the zero bit and  $x$ . Formally in LNS, a number  $X$  is represented as the triplet

$$X_{LNS} = (Z_x, S_x, X)$$

Due to the basic properties of the logarithm, the multiplication of  $X_{LNS}$  and  $Y_{LNS}$  is reduced to the computation of the triplet  $Z_{LNS}$ ,

$$Z_{LNS} = (Z_z, S_z, Z)$$

Where,  $Z_z = z_x \vee z_y$ ,  $S_z = a_x \oplus a_y$ , and  $Z = x + y$ . Similarly the case of division reduces to binary subtraction. The derivation of the logarithm  $a$  of the sum  $A$  of two triplets is more involved, as it relies on the computation of

$$a = \max\{x, y\} + \log_b(1 + b^{-|x-y|}) \\ = \max\{x, y\} + \phi_a(d),$$

where  $\phi_a(d) = \log_b(1 + b^{-d})$  and  $d = |x - y|$

Similarly the derivation of the difference of two numbers, requires the computation of

$$c = \max\{x, y\} + \log_b(1 - b^{-|x-y|}) \\ = \max\{x, y\} + \phi_s(d),$$

Where  $\phi_s(d) = \log_b(1 - b^{-d})$ . Assume that a two's-complement (TC) word is used to represent the logarithm  $x$ , composed of a  $k$ -bit integral part and an  $l$ -bit fractional part. The range  $D_{LNS}$  spanned by  $x$  is given by

$$D_{LNS} = [-b^{2^{k-1}-2^{-1}}, -b^{2^{-1}}] \cup \{0\} \cup [b^{2^{-1}}, b^{2^{k-1}-2^{-1}}]$$

to be compared with the range of  $(-2^{i-1}, 2^{i-1} - 2^{-f})$  of a linear TC representation of  $i$  integral bits and  $f$  fractional bits. In general, LNS offers a superior range, over the linear two's-complement representation. This is achieved using comparable word lengths, by departing from the strategy of equispaced representable values and thus resorting to a scheme that resembles floating-point arithmetic. The basic organization of an LNS adder & subtractor is shown in Fig.2. The parallel subtractions

$$s_1 = x - y \\ s_2 = y - x$$

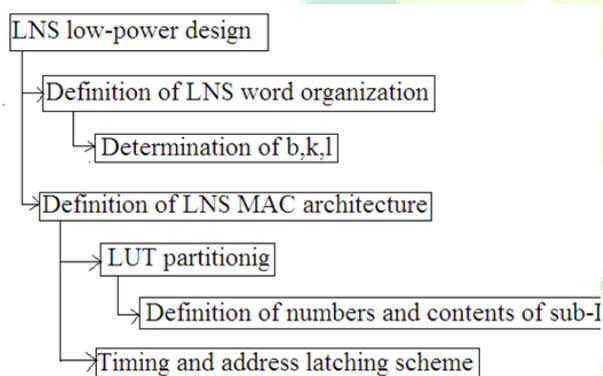
are implemented, followed by a multiplexer, which computes  $d$  according to the rule

$$d = |x - y| = \begin{cases} s_1, & s_1 > 0 \\ s_2, & \text{otherwise} \end{cases}$$

### III. PROPOSED DESIGN METHODOLOGY

This project presents a low-power design framework for LNS-based systems,

composed of several techniques. We quantify the impact of the constituent design techniques using detailed simulations of the derived LNS circuits. The proposed design framework is depicted in Fig.1. Initially, optimal selection of LNS design parameters is sought, including word length and base assuming simple partitioned look-up table architecture. Subsequently, in the second stage of the proposed framework design techniques and the derived architectures are presented, targeting LNS multiply-accumulate (MAC) units in 180nm technology. To illustrate the use of the proposed framework, the area-time-power design space of a low-pass FIR filter is explored for several configurations of MAC units. The proposed study focuses on the use of partitioning as a technique to limit the exponential growth of the size of LUTs with the word length. The technique is simple and leads to fast circuits.



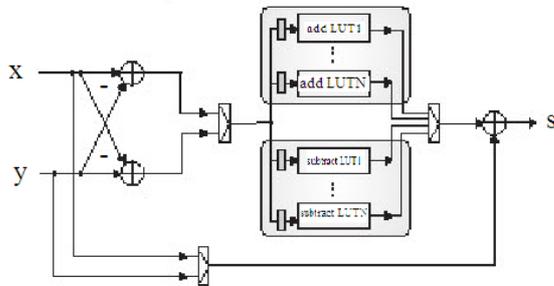
**Fig.1. Proposed Design Methodology**

Departing from direct look-up table organization, a variety of LNS architectures

for addition, subtraction, and multi-operand operations, have been proposed in the literature employing interpolation, linear or polynomial approximation aiming at reducing the memory requirements, particularly for larger word lengths, such as 32 bits. These ideas have been combined with mathematical decompositions and transformations of the basic operations, exploiting the particular characteristics of the functions to further simplify approximation.

#### IV. LOW-POWER DESIGN OF LNS CIRCUITS

In the proposed design framework, power dissipation reduction is sought by partitioning the particular LUTs into smaller LUTs, called sub-LUTs, only one of which is active per operation. This organization is shown in Fig.4.2. To guarantee that no dynamic power is dissipated in the inactive sub-LUTs, the corresponding sub-LUT addresses are latched and remain constant throughout a particular operation. Further power dissipation reduction is sought at the implementation of multiply-accumulate units, by using retiming techniques, as well as at the algorithmic optimization level by judiciously selecting the parameters of the LNS representation.



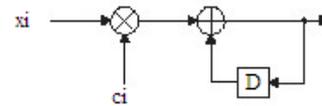
**Fig.2. The organization of an LNS adder and subtractor**

Complexity reduction in LNS processors by partitioning of the LUTs has been successfully applied in the past. Here we focus on combinational logic implementation of LUTs, instead of memory-based implementation. The organization of the LNS adder/subtractor comprises N sub-LUTs per operation, as shown in Fig.4.2. The upper sub-LUT system corresponds to function  $\phi_a$  (d) required for LNS addition, i.e., addition of operands having the same sign, while the lower sub-LUT system is used for LNS subtraction, i.e., addition of operands of different signs.

## V. LNS MAC ARCHITECTURES

The basic structure of the single-MAC unit is shown in Fig.3. Symbols  $\otimes$  and  $\oplus$  denote a multiplier and an adder respectively, while D denotes a delay unit, implemented as a register. The study is extended to the performance of a two-MAC and four MAC architecture, shown in Figs.4 and Fig.5 respectively.

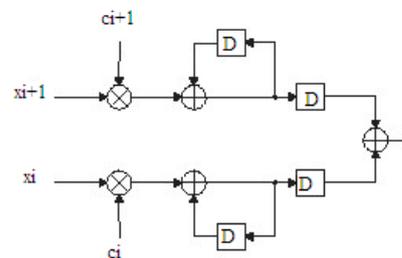
An FIR filter is described by,



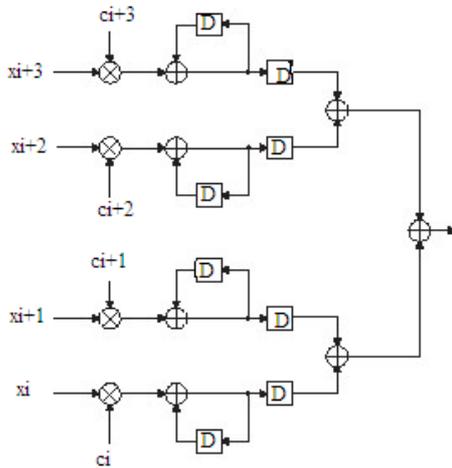
**Fig.3. Organization of single MAC Architecture**

$$y(n) = \sum_{j=0}^{N-1} C_j X(n - j)$$

Where  $C_i$  are the filter coefficients,  $X(n)$  is the input sequence and  $Y(n)$  is the output sequence. In order to map the computation into two-MAC architecture, computation is partitioned into the computation of two terms, subsequently added, as follows



**Fig.3. Organization of two MAC Architecture**



**Fig.3. Organization of four MAC Architecture**

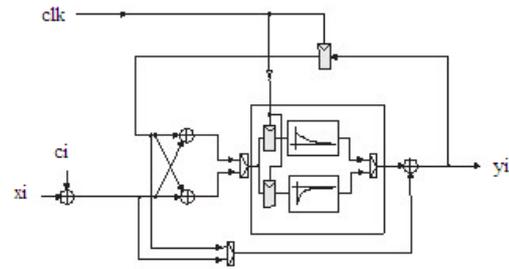
$$y(n) = \sum_{j=0}^{\lfloor \frac{N}{P} \rfloor - 1} X(n - 2j) C_{2j} + \sum_{j=0}^{\lfloor \frac{N}{2} \rfloor - 1} X(n - 2j - 1) C_{2j+1}$$

Each term is allocated to a MAC of two-MAC architecture. In general, for a P-MAC architecture, the computation is decomposed as

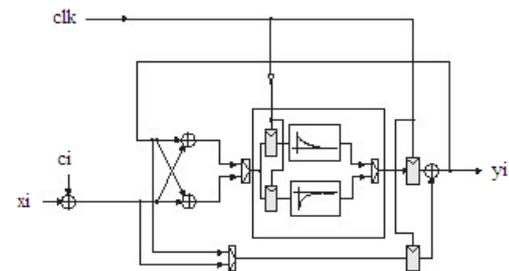
$$y(n) = \sum_{j=0}^{\lfloor \frac{N}{P} \rfloor - 1} X(n - Pj) C_{Pj} + \dots + \sum_{j=0}^{\lfloor \frac{N}{P} \rfloor - 1} X(n - Pj - P - 1) C_{Pj+P-1} = \sum_{p=0}^{P-1} \sum_{j=0}^{\lfloor \frac{N}{P} \rfloor - 1} X(n - Pj - p) C_{Pj+p}$$

The pth MAC unit computes  $S_p = X(n - Pj - p) C_{Pj+p}$ . Subsequently, the summation of all  $S_p$  is computed. The LNS equivalent of a single-MAC architecture is depicted in Fig.6, where the binary multiplier has been

replaced by an adder, and the binary adder is mapped to an LNS adder/subtractor.



**Fig.6. LNS MAC unit**

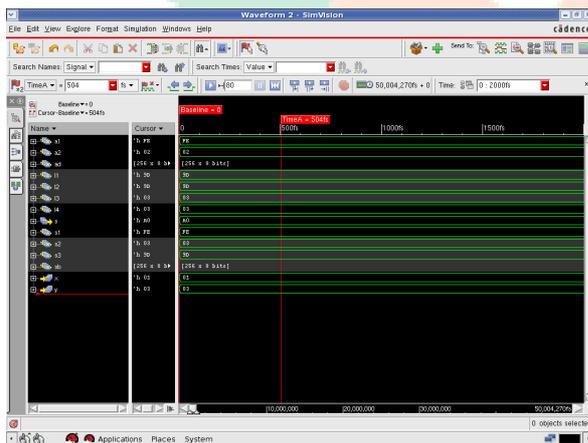


**Fig.7. Retimed LNS MAC unit**

In the implementation of Fig.6, it is evident that the paths to the inputs of the final adder are not balanced, thus leading to excessive switching activity at the adder following the memory structure. The amount of the switching activity depends on the logic depth of the LUT implementation. A solution to this problem is to retime the circuit so that the register located at the feedback path is replaced by registers placed at the inputs of the final adder, as shown in Fig.7. Power dissipation simulations have shown that the retimed circuit is more efficient. Hence, the retimed circuit is adopted for the LNS MAC implementations.

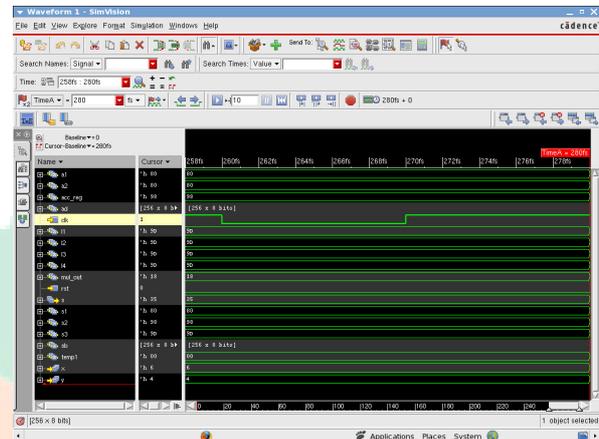
## VI. RESULTS AND DISCUSSION

The LNS adder & subtractor is simulated by using Cadence Tool as shown in Fig.8., X and Y inputs are added and subtracted with the X-Y and Y-X respectively. Addition or subtraction values are selected using MUX and the values are taken from the look-up table(LUT) and the waveform shows the output S.



**Fig.8. Simulation Result for LNS adder & subtractor using Cadence**

The LNS MAC Architecture is simulated by using Cadence Tool as shown in Fig.9., the LNS is implemented using the MAC unit. X and Y inputs are added and subtracted with the X-Y and Y-X respectively. Addition or subtraction values are selected using MUX and the values are taken from the look-up table(LUT) and the waveform shows the output S.



**Fig.9. Simulation Result for LNS MAC Architecture using Cadence**

## VII. CONCLUSIONS

This project quantitatively shows that the adoption of LNS can lead to very efficient circuits for digital filtering applications when appropriately selecting the logarithmic base and the wordlength in a contemporary 180nm technology. An LNS-based system using the proposed adder & subtractors offers substantial power dissipation savings at no performance penalty. Partitioning of the LUTs is employed to create parts in the circuit that can be independently activated thus reducing power dissipation. Power has been reduced by latching the inputs to the LUTs.

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