



# Controller Tuning for Disturbance Rejection Associated with Delayed Double Integrating Processes, Part VI: PI-P Controller

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**Abstract**— This research paper deals with investigating the possibility of using a PI-P controller for the purpose of disturbance rejection associated with delayed double integrating processes. The MATLAB optimization toolbox is used to tune the PI-P controller parameters to achieve optimal performance of the linear control system excited by process unit step disturbance. Five objective functions based on the time response error of the control system are used, and the best function is assigned. The simulation results show the suitability and robustness of the PI-P controller for process delay time up to 20 seconds. The ITSE objective function is appropriate for use with the PI-P controller and the delayed double integrating process. It is possible to go with the maximum time response to a very low levels with a fast time response. The studied controller is capable of competing with other types used in this series of papers except with the PI-PD controller.

**Index Terms**— PI-P controller tuning, disturbance rejection, delayed double integrating process, control system performance, MATLAB optimization toolbox.

## I. INTRODUCTION

This is the six paper in a series of research papers investigating the use of specific controllers to control the disturbance rejection associated with delayed double integration processes. The PI-P controller is known from some time and used for reference input tracking. This is the first time to investigate deeply the possibility of using the PI-P controller for disturbance rejection.

Lee, Taylor, Chotai, Young and Chalabi (1996) designed a robust PIP controller for temperature, humidity and Carbon dioxide control in a glasshouse. They claimed that using PIP controller showed tight control to desired setpoints for the three variables of the glasshouse [1]. Dixon, Young, Chotai and Shao (1996) discussed the design and implementation of an optimal PIP controller for a large inverted pendulum system. They used a data-based model of the unstable non-minimum phase open-loop system for linear quadratic optimal design of the PIP controller [2]. Taylor, Chotai and Young (1998) discussed the robustness and disturbance response characteristics of two PIP control structures through simulation examples and the design of a climate control system for a large horticultural glasshouse system [3].

Chotai, Young, McKenna and Tych (1998) extended the PIP controller for SISO linear system to MIMO systems. The PIP control law exploited the full power of state variable feedback control within non-minimum state space settling. They illustrated the effectiveness of using the PIP controller by simulation and practical examples [4]. Taylor, Chotai and Young (2001) provided an introduction to the non-minimum state space / PIP control design methodology and associated system identification procedure. They illustrated the application of their technique using the IFAC93 benchmark system [5]. Heying, Baohong and Wenhua (2003) proposed an active queue management algorithm based on using a proportional-integral and position feedback compensators (PIP). They claimed that the PIP could eliminate the error incurred by the inaccuracy in the linear system model and eliminates the sensitivity to the changes in the system parameters [6].

Al-Hammouri, Liberatore, Branicky and Philips (2006) designed a technique to find the stability regions of PI and PID controllers for TCP AQM. They showed that the PIP controller can be unstable in the presence of delays even for the control parameters given in the literature [7]. Ozbay and Gundes (2008) derived a simple design procedure for a restricted class of plants satisfying the PIP and at most two real blocking zeros in the extended right half plane. There was no restriction on the number and location of the poles and on the number of the left half plane zeros. The controller order did not exceed that of the plant [8]. Somasundaram and Bhaba (2010) proposed a coefficient diagram method based PI-P control strategy to operate a bio-reactor effectively at unstable steady-state condition. The reactor was defined by an unstable first-order plus time delay model. They compared the proposed strategy with other control strategy [9].

Legweel, Lazic, Ristanovic and Sajic (2014) investigated using PIP cascade control to acquire better performance in the central air conditioning system. They showed that the PIP cascade controller had the capability of self-adapting to system changes and resulted in faster response and better performance [10]. Ruiza, Jimenez, Sanchez and Domidob (2014) addressed the design task of PI-P event-based controllers through the use of interactive tools. They

demonstrated the usefulness of the analyses used through several designs for typical industrial examples [11].

## II. PROCESS

The process is a delayed double integrating process having the transfer function,  $G_p(s)$  [12]:

$$G_p(s) = (K_p/s^2) \exp(-T_d s) \quad (1)$$

Where:

$K_p$  = process gain.

$T_d$  = process time delay.

To facilitate the dynamic analysis of the control system incorporating the controller and process, the exponential term in Eq.1 has to be written in a polynomial form. The simplest form of which is the first-order Taylor series [13]. Using the first-order Taylor series for the exponential term, Eq.1 becomes:

$$G_p(s) = (-K_p T_d s + K_p) / s^2 \quad (2)$$

## III. CONTROLLER

The controller is a Proportional Integral Plus Proportional (PI-P) Controller. It has the structure shown in Fig.1 for a control system with a reference input [7], [9].

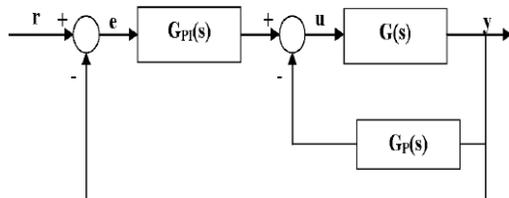


Fig.1. PI-P controller structure [7],[9].

With disturbance input associated with the process, the block diagram of the control system has two inputs: reference input  $R(s)$  and disturbance input  $D(s)$ . The new block diagram of the control system with the two inputs is shown in Fig.2.

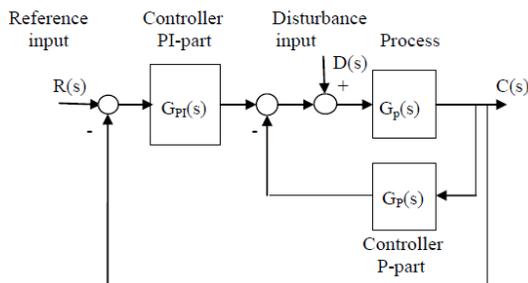


Fig.2. Control system block diagram with two inputs.

The PI-P controller has two parts:

Part I: Feedforward PI-part having a proportional + integral controller structure. It has the transfer function,  $G_{PI}(s)$

given by [9]:

$$G_{PI}(s) = K_{pc} [1 + 1/(\tau_i s)] \quad (3)$$

Where:

$K_{pc}$  = proportional gain of the PI-controller.

$\tau_i$  = integral time constant of the PI-controller.

Part II: Feedback P-part having a proportional controller structure. It has the transfer function,  $G_P(s)$  given by [9]:

$$G_P(s) = K_f \quad (4)$$

Where:

$K_f$  = gain of the feedback proportional controller.

This means that the PI-P controllers has three parameters to be tuned for proper performance of the control systems. Those are the proportional gain  $K_{pc}$ , the integral time constant  $\tau_i$  and the feedback proportional gain  $K_f$ .

## IV. CONTROL SYSTEM TRANSFER FUNCTION

For purpose of this study is to investigate the PI-P controller effectiveness in disturbance rejection of the delayed double integrating process. In this context, the control system input will be the disturbance  $D(s)$  and the output is the process output  $C(s)$ . The reference input in the block diagram of Fig.2 [ $R(s)$ ] will be set to zero. In this case, the closed loop transfer function  $M(s)$  of the closed-loop system will be:

$$M(s) = C(s)/D(s) = (b_0 s^2 + b_1 s) / (a_0 s^3 + a_1 s^2 + a_2 s + a_3) \quad (5)$$

Where:

$$b_0 = -K_p T_d \tau_i$$

$$b_1 = K_p \tau_i$$

$$a_0 = \tau_i$$

$$a_1 = -K_p T_d (K_{pc} + K_f) \tau_i$$

$$a_2 = K_p (K_{pc} + K_f) \tau_i - K_p K_{pc} T_d$$

$$a_3 = K_p K_{pc}$$

## V. PI-P CONTROLLER TUNING

The PI-P controller is tuned as follows:

1. An error function is defined as the difference between the time response of the control system  $c(t)$  and a desired value. The desired value for disturbance rejection purpose is zero. Therefore, the error function is:
 
$$e(t) = c(t) \quad (6)$$
2. An objective function is assigned to be minimized by a multi-dimensional optimization technique. Here, five objective functions are investigated which are the ITAE, ISE, IAE, ITSE and ISTSE [14] to [16].
3. The MATLAB optimization toolbox is used to minimize the objective functions yielding the three tuned controller parameters using its command 'fminuc' [17].
4. The MATLAB control toolbox is used to plot the step

time response of the control system to a unit disturbance input using the command 'step' and extract some of the time based characteristics of the control system using the 'stepinfo' command [18].

5. The time delay of the process is changed in a range governed by the possibility of obtaining a stable unit disturbance time response.
6. A MATLAB code was written by the author to apply the mentioned steps.
7. A sample of the code outputs for the five objective functions ITAE, ISE, IAE, ITSE and ISTSE is given in Table 1 for a 0.1 s time delay and a unit process gain.

TABLE I: TUNED CONTROLLER PARAMETERS AND CONTROL SYSTEM CHARACTERISTICS

	ITAE	ISE	IAE	ITSE	ISTSE
$K_{pc}$	111.0286	32.5090	110.6410	122.6071	112.9235
$K_f$	67.3561	4.6498	29.0740	101.9542	84.3854
$\tau_i$ (s)	0.3039	1.0966	0.4184	0.2604	0.2746
$c_{max}$	0.0066	0.0377	0.0087	0.0051	0.0059
$T_{cmax}$ (s)	0.3434	0.6149	0.3672	0.3267	0.3280
$T_s$ (s)	0	0	0	0	0

8. The ITSE objective function resulted in obtaining the best performance of the control system for disturbance rejection. Therefore, it was considered for the rest of the process time delay.
9. The settling time is evaluated as the time after which the time response due to a unit disturbance input stays within a value of  $\pm 0.05$ .
10. The values of the maximum time response  $c_{max}$ , time of maximum response  $T_{cmax}$  and settling time  $T_s$ , all indicate the effectiveness of using the PI-P controller for disturbance rejection.
11. The time response of the control system due to a unit disturbance input for an 0.1 s time delay is shown in Fig.3 for the five objective functions.

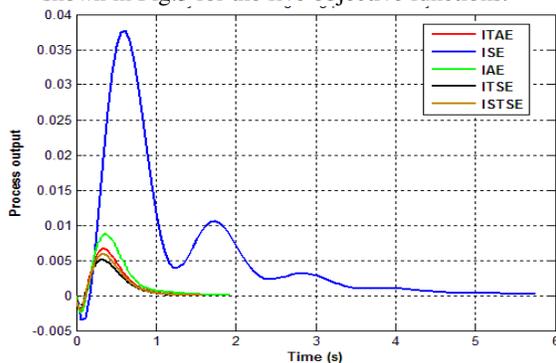


Fig.3. Effect of objective function on disturbance time response.

12. The effect of the process time delay on the time response of the control system using the ITSE

objective function for a unit gain double integrating process is shown in Fig.4 for a time delay between 0.2 and 1 s.

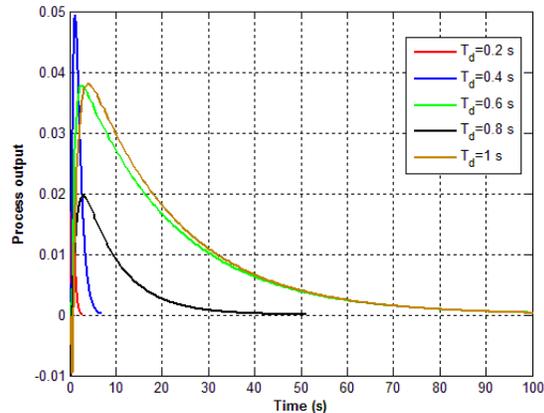


Fig.4. Effect of process time delay on system time response for time delay  $\le 1$  s.

13. The effect of time delay on the time response of the control system due to unit disturbance input is shown in Fig.5 for a process delay time of 5, 10 and 15 s.

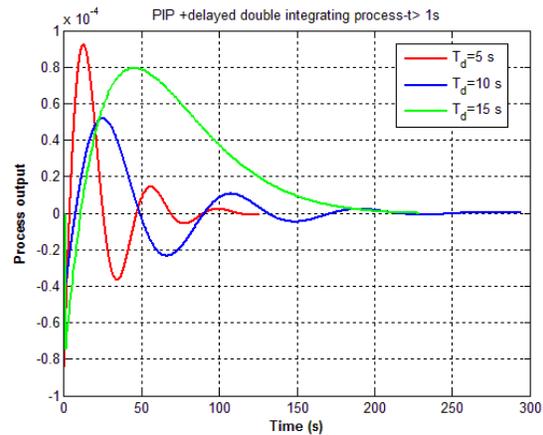


Fig.5. Effect of process time delay on system time response for 5, 10, 15 s time delay.

14. The maximum value of the time response to a unit step disturbance input is used as quantitative measure of the effectiveness of the controller used for disturbance rejection. Fig.6 shows the effect of the process time delay on the maximum time response for a delay time range up to 20 seconds.

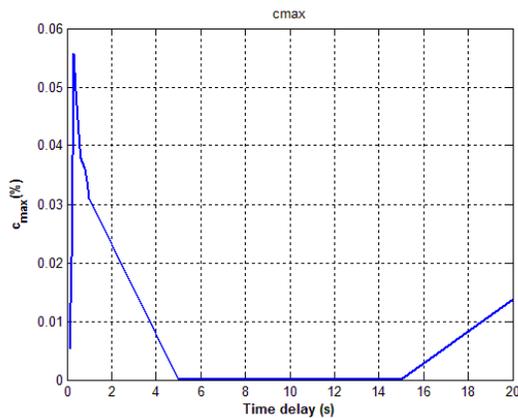


Fig.6. Effect of process time delay on the maximum time response.

15. The time of the maximum time response to a unit step disturbance input is used as quantitative measure of the effectiveness of the controller used for disturbance rejection in terms of the response speed. Fig.7 shows the effect of the process time delay on the time of the maximum response for a delay time range up to 20 seconds.

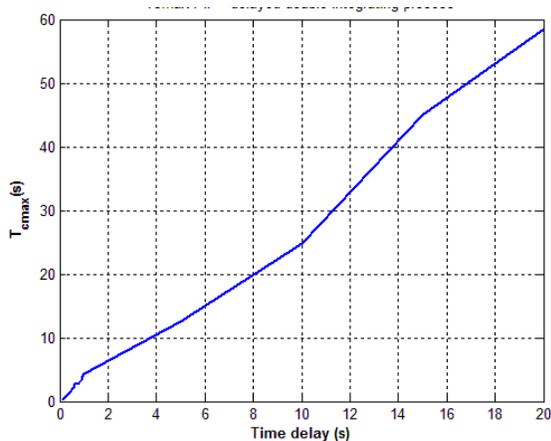


Fig.7. Effect of process time delay on the time of maximum response.

## VI. COMPARISON WITH OTHER RESEARCH WORK

To judge the effectiveness of using the PI-P controller for disturbance rejection associated with delayed second order processes, it has been compared with some other controllers used with the same process.

Fig.8 shows a comparison between the time response of the control system during disturbance rejection using three different controllers: I-PD [19], PI-PD [20] and PI-P (present) controllers. The effectiveness of the PI-P controllers comes between the I-PD and PI-PD controllers.

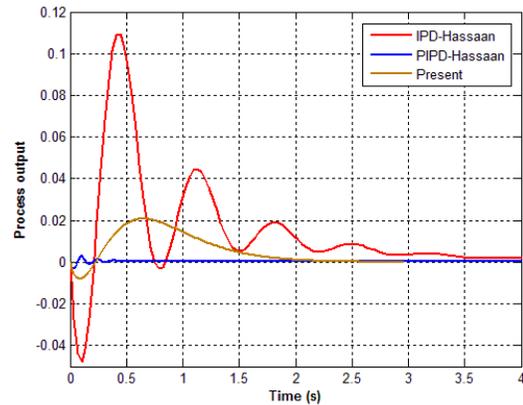


Fig.8. Time response comparison for 0.2 s time delay.

## VII. CONCLUSION

- The possibility of using a PI-P controller for disturbance rejection associated with delayed double integrating processes was investigated.
- The controller was tuned to adjust its three parameters for optimal performance using five objective functions and the MATLAB optimization technique.
- The best objective function was assigned which was the ITSE one.
- The effect of process time delay on the disturbance time response was investigated for time delay up to 20 seconds.
- The PI-P controller could generate disturbance response of low levels up to 0.055 and time of maximum disturbance time response of about three times the process time delay.
- It succeeded to provide disturbance time response of zero settling time for the time delay range investigated.
- The PI-P controller was robust with respect to the variation in the process time delay.
- The performance of the closed loop control system for disturbance rejection using a PI-P controller was compared with that using I-PD and PI-PD controllers for the same purpose.
- The PI-P controller could compete with all the other types of controllers studied in this series of research papers except with the PI-PD controller.

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### BIOGRAPHY



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<http://scholar.cu.edu.eg/galal>

### DEDICATION



- I dedicate this research work to my friend Mr. Nabil Gharip.
- Mr. Gharip is a General Manager in the Transformer Electricity Company, Bani Sweif, Egypt.
- I have known Mr. Gharip only few months ago, but I feel that I know him from 10's of years ago.
- I am happy to know him and consider him as an intimate friend.
- His deeds show how great and useful he is for his community and society
- This is why I dedicate this work to him.