

# Cultivation of Mathematical Problem Awareness among High School Students under Core Literacy

Yifan Li, Liang Fang\*

College of Mathematics and Statistics, Taishan University, Tai'an, China

\*Email:fangliang3@163.com

**Abstract**—With the continuous advancement of education reform, cultivating students' core literacy has become a trend in education. An important way to achieve core literacy is to improve problem awareness, which has become the driving force for promoting social progress. It can be seen from this that it is necessary to cultivate high school students' awareness of mathematical problems.

At present, most teaching methods adopt a transmission based approach, with teachers pursuing teaching progress and directly teaching students to improve teaching efficiency. Students are in a passive position, so the research on student problem awareness under core literacy is worthy of attention.

This article first introduces the relevant concepts of mathematical problem awareness. Then, introduce the current situation of high school students' awareness of mathematical problems and analyze it. Finally, specific suggestions are put forward for cultivating students' awareness of mathematical problems under the core literacy. This article aims to cultivate high school students' awareness of mathematical problems, help them build a knowledge system, and lay a solid foundation for the learning of high school mathematics.

**Index Terms**—Mathematical core literacy, mathematical problem awareness, constructivist learning theory, zone of proximal development theory, self-efficacy theory.

## I. INTRODUCTION

Einstein once said, "Asking a problem is more important than solving it." [1-5] It can be seen that the ability to ask questions is something we should focus on cultivating. In fact, the improvement of problem awareness can not only promote social development, but also advance education. Discovering problems is one of the good qualities in learning. Only when students engage in in-depth and thorough thinking during their studies can they develop problems, thus generating learning motivation to explore methods and approaches to solving problems. Through the process of discovering problems, understanding problems, and solving problems, they can gain the truth behind them. However, cultivating problem awareness is a long-term and complex process that not only requires learners to self regulate from within, but also requires educators to apply certain external forces to guide learners. The high school stage is the most important stage in a student's learning career, which requires teachers to attach great importance to the cultivation of mathematical problem awareness among high school students.

## II. THE RESEARCH SIGNIFICANCE OF HIGH SCHOOL STUDENTS' MATHEMATICAL PROBLEM AWARENESS UNDER CORE LITERACY

### A. Help students enhance their awareness of mathematical problems under core competencies

Improving students' awareness of mathematical problems helps them discover problems in their learning and daily life. When encountering problems, they can deeply think about them, integrate new situations in the problem with their existing knowledge structure, and engage in in-depth thinking. This process exercises students' logical abstraction, thinking ability, etc., which helps to further develop their mathematical problem awareness.

### B. Provide strategies for cultivating mathematical problem awareness for teachers

Teachers may encounter various complex situations during the teaching process. For experienced teachers, they can adjust teaching methods and strategies in a timely manner through these situations to improve students' mathematical problem awareness. However, for some new teachers, due to their lack of experience, they cannot handle these problems well when facing complex teaching situations. Therefore, cultivating students' mathematical problem awareness through research can broaden the perspective of novice teachers. In practice, some suggestions can be provided to cultivate students' problem awareness, so that novice teachers can choose teaching methods and means reasonably in teaching, and promote the cultivation of students' mathematical problem awareness in teaching.

### C. Helps to adapt to the needs of social development

In today's rapidly developing social context, innovation ability is a highly valued ability in various industries and fields. Mathematical problem awareness can stimulate students' curiosity and exploration desire, encourage them to explore new ideas and methods, and better cultivate innovative thinking. Therefore, students with strong awareness of mathematical problems have higher levels of thinking ability, creativity, and other mathematical abilities, which can greatly promote social development. The research on the mathematical problem awareness of

high school students under the core literacy helps to cultivate more talents with innovative spirit and practical ability, and contribute to the development of society.

### III. CURRENT SITUATION AND CAUSE ANALYSIS OF MATHEMATICS PROBLEM AWARENESS AMONG HIGH SCHOOL STUDENTS

#### *A. Current Situation of Mathematics Problem Awareness among High School Students*

##### (1) Lack of proactive exploration by students

In the context of exam oriented education, students often only focus on scores and grades, using rote memorization and mechanical training to cope with exams, neglecting a deep understanding and exploration of knowledge. Many high school students exhibit a passive learning attitude in the process of learning mathematics, relying on teacher explanations and textbook guidance, only focusing on theories and formulas in textbooks, and lacking sensitivity to mathematical problems in daily life. When encountering mathematical problems, they lack the awareness of linking the problem with reality. They often choose to wait for the teacher's answer, rather than actively seeking learning resources and problem-solving methods. This passive learning method is not conducive to cultivating students' awareness of mathematical problems. Students feel bored and bored during the learning process, making it difficult to stimulate their interest and motivation in learning.

##### (2) Lack of confidence among students

Due to the complexity and abstraction of high school mathematics knowledge, many high school students may develop fear or irritability when learning mathematics, thinking that they find it difficult to understand high school mathematics knowledge, especially for students with weak mathematical foundations in middle school. They may find it difficult to adapt to the high-intensity and difficult learning of high school mathematics, and feel overwhelmed with difficulties in high school mathematics learning. This can lead to doubts about their learning ability in mathematics, seriously undermining their confidence and enthusiasm in learning mathematics. In addition, during high school, students face pressure from various aspects such as further education and family. In this environment, some students may have doubts about their abilities, especially in mathematics, a subject that requires high thinking ability, thinking that they cannot master mathematical knowledge and lack confidence in learning mathematics well.

##### (3) Uneven learning abilities of students

Although high school mathematics education focuses on cultivating students' problem-solving abilities, the actual results are not ideal. Some students are able to apply their knowledge to logical reasoning and calculations

when solving mathematical problems, but there are still a considerable number of students who feel at a loss when facing problems. Some students are good at summarizing and generalizing, able to apply their learned knowledge to practical problem-solving, while others may lack this ability and only mechanically memorize and imitate; Some students have a solid grasp of basic knowledge, strong coherence of knowledge, and can quickly find the starting point and correct answers when solving problems, while others may be unable to start solving problems due to weak basic knowledge; Some students are passionate about mathematics and willing to invest time and energy in learning and exploring, while others lack this interest and motivation, only passively accepting knowledge and finding it difficult to take initiative in the learning process. The imbalance in problem-solving ability reflects the differences in mathematical problem awareness among students.

##### (4) Students lack critical thinking

Critical thinking is an important component of mathematical problem awareness. However, many high school students often lack critical thinking when solving mathematical problems. They often only accept the textbook and the teacher's perspective, are superstitious about the teacher and the knowledge in the textbook, lack questioning and reflection on the solution of mathematical problems, blindly worship authority, and make it difficult for students to actively discover errors and loopholes. On the other hand, during the teaching process, teachers play the role of educators and are in a dominant position, making it difficult for students to have the opportunity to actively think and question. Students often hesitate to ask their own questions, although they have doubts in their hearts. On the one hand, they are afraid that the teacher will ignore their own questions, and on the other hand, they are worried that the questions they ask are too simple, and may even attract ridicule from their classmates, which can undermine their confidence. This lack of critical thinking phenomenon makes it difficult for students to form independent mathematical insights. Over time, their critical thinking ability will gradually deteriorate, making it difficult to make significant breakthroughs in mathematics learning.

##### (5) Insufficient understanding of mathematical problem awareness among teachers

With the increasing demand for innovative talents in society, the education department in China attaches great importance to cultivating problem awareness. The research on problem awareness in various schools in our country is still in its early stages. Although new teaching models have been proposed in teaching, students are rarely placed in the main position in the classroom. In the current teaching classroom, there is a certain gap between the

teaching philosophy of teachers and modern teaching philosophy. At present, most teaching modes are: students think and solve the problems raised by teachers, students are in a passive position, and everything is led by the teacher's nose. In order not to affect the teaching progress, teachers even teach knowledge throughout the entire class, leaving little time for students to think about problems. Students can only adapt to the teaching mode of the teacher, which is not conducive to the cultivation of their problem awareness and the improvement of their ability to raise and solve problems.

#### *B. Cause analysis*

##### *(1) The impact of the education system*

The education system is the cornerstone of a country or region's educational development and has a profound impact on various aspects of students. Mathematical problem awareness, as one of the core elements of learning mathematics, is directly or indirectly influenced by the education system. The new curriculum under the core literacy advocates self-directed exploration and cooperative learning as the main learning methods. But under the command of the "baton" of the college entrance examination, the current education system places too much emphasis on exam oriented education, which leads to students lacking in-depth thinking on mathematical problems. Students under the pressure of exam oriented education often only focus on exam scores and grades, neglecting the essence and connotation of mathematical problems. This will lead to students lacking initiative and creativity in mathematics learning, making it difficult to form clear problem awareness. Over time, students lose interest in asking questions.

The curriculum design in the education system is the starting point for students to access mathematical knowledge. A reasonable curriculum design can build a systematic mathematical knowledge system for students, while also stimulating their awareness of mathematical problems. In addition, the difficulty gradient of the curriculum can also affect students' awareness of mathematical problems. Overly simple courses cannot stimulate students' desire to challenge, while overly complex courses may deter students and make it difficult to form a positive awareness of mathematical problems.

The evaluation and feedback mechanism is an important component of the education system, which has a significant impact on cultivating students' awareness of mathematical problems. Under exam oriented education, academic performance represents everything. This one-sided evaluation method leads students to blindly pursue academic performance, neglecting the thinking and solving of mathematical problems, which is not conducive to talent cultivation. Therefore, diversified evaluation and feedback mechanisms are conducive to cultivating

students' awareness of mathematical problems.

##### *(2) Limitations of teaching methods*

As a core link in the educational process, teaching methods have a profound impact on the learning outcomes, thinking patterns, and problem awareness of students. However, in some cases, improper use of teaching methods may have a negative impact on students' awareness of mathematical problems.

Most middle schools use lecture based teaching, with teachers taking the lead and students passively accepting it. Teachers directly impart knowledge to students during the teaching process, with little or no emphasis on guiding their problem awareness. The advantage of lecture based teaching is to improve teaching efficiency, which can impart a large amount of knowledge and experience to students. However, due to passive thinking, it is not conducive to cultivating students' awareness of mathematical problems. In this mode, students often lack opportunities to think and ask questions, leading to a decrease in their sensitivity and initiative towards mathematical problems. Over time, their awareness of mathematical problems will gradually weaken, making it difficult to form effective mathematical problem awareness.

In some teaching methods, teachers often overlook individual differences among students and adopt a one size fits all teaching approach. Different students have different knowledge backgrounds, cognitive styles, and learning abilities, so this approach not only fails to meet the needs of different students, but may also cause some students to feel confused and frustrated, thereby reducing their awareness of mathematical problems.

##### *(3) The impact of individual differences among students*

There are differences in thinking patterns and interests among individual students, which leads to differences in their awareness of mathematical problems. Some students may have strong mathematical thinking and problem-solving abilities, and can actively solve mathematical problems; However, other students may lack these abilities, leading to insufficient performance in mathematical problem awareness.

##### *(4) Lack of training opportunities for problem posing*

In the traditional classroom teaching of high school mathematics, almost the entire class is taught by the teacher, and during the learning process, students are also passively following the teacher's thinking. If students encounter problems during class and cannot understand them in a timely manner, it will affect the next stage of listening. In addition, in the classroom, teachers mainly deal with problems in textbooks and homework exercises, lacking understanding of cultivating students' thinking abilities, which greatly restricts the formation of problem awareness among students.

#### IV. PRINCIPLES AND STRATEGIES FOR CULTIVATING HIGH SCHOOL STUDENTS' AWARENESS OF MATHEMATICAL PROBLEMS

##### *A. Principles for cultivating high school students' awareness of mathematical problems*

###### (1) Principle of gradual progress

The principle of gradual progress refers to starting from the order of students' physical and mental development, and following certain steps and sequences step by step. The principle of gradual progress requires teachers to make reasonable arrangements for teaching content and methods, and to teach step by step in order, in accordance with the laws of students' understanding of things. We cannot ignore or cross a certain stage and directly proceed to the next stage of teaching, so that students have a comprehensive grasp of the knowledge structure, their abilities are fully developed, and their mathematical problem awareness is developed and improved.

###### (2) Principle of integrating theory with practice

The principle of linking theory with practice refers to combining theoretical knowledge with practical situations. For student learning, combining theory with practice can connect strong theoretical knowledge with practice, help students understand and construct knowledge well during the learning process, and also improve their ability to apply newly learned knowledge to solve problems in daily life.

Teachers should make sufficient lesson preparation before class, guide students to apply theoretical knowledge to practical problems during teaching, and add practical activities in a timely manner to improve students' understanding and development ability, help students understand the practical significance and application value of knowledge, and enable them to learn and grow in practice. In short, the principle of integrating theory with practice can promote the organic integration of theory and practice, improve students' learning efficiency, facilitate the development of their knowledge, and promote social progress.

###### (3) Principle of individualized teaching

Due to each student having their own living environment, growth characteristics, and inner world, each student is unique. During teaching, teachers should make overall planning and systematic arrangements for the teaching process based on individual differences among students, using appropriate teaching methods and means to stimulate their learning potential, enable them to maximize their abilities, and achieve optimal development.

The principle of teaching students according to their aptitude reflects the concept of educational equity. It can not only help each student to leverage their strengths,

overcome their weaknesses, improve their overall quality, and achieve comprehensive development, but also enable teachers to better understand students, prescribe targeted solutions, enhance their awareness of mathematical problems, and promote development.

###### (4) Principle of Incentives

The motivational principle is to awaken students' learning motivation through certain teaching methods, enabling them to spontaneously engage in learning. The principle of motivation can be achieved through verbal and behavioral incentives, with the aim of enhancing students' cognitive intrinsic drive, from the inside out, and ultimately promoting their comprehensive development.

When teaching, teachers should pay attention to the timing and methods of motivation, and the timing of motivation should be just right, neither too frequent nor too sparse. They should use positive language as much as possible, create a harmonious, positive, and joyful atmosphere, and allow students to achieve success and a sense of achievement in the process of solving problems; Teachers can also organize various forms of mathematical games to allow students to showcase their mathematical talents in competition and cultivate their competitive awareness. Moderate competition can help students build self-confidence, overcome difficulties and setbacks in learning, enhance their interest and motivation in learning, and thus more actively engage in mathematics learning; When evaluating students, it is important to take the progress they have made on their existing foundation as a reference frame, so that they can not only learn knowledge but also experience the joy of learning.

##### *B. Strategies for cultivating high school students' awareness of mathematical problems*

###### (1) Teacher's problem awareness should be targeted and purposeful

Based on the background of core competencies, "How to effectively cultivate students' awareness of raising questions and their ability to solve problems".

This issue has attracted more and more attention from teachers [2]. As teachers, we should be aware that the purpose of raising questions in classroom teaching should be accurate and clear, and should not exceed the scope of students' abilities, in line with their cognitive characteristics. A clear purpose can stimulate students' awareness of participating in the classroom, allowing them to have questions and answers. Otherwise, it becomes a one-sided teaching where teachers ask and answer. To explore problems in a student-centered learning model led by teachers. In addition, teachers should pay attention to the creativity, innovation, and connection with practical problems when proposing problems, so that students will find joy in learning through exploration.

###### (2) Teachers should fully prepare for lessons



Due to the complexity of the teaching process, teachers may encounter various situations during teaching. Experienced teachers can adapt and handle them flexibly. For new teachers, sufficient lesson preparation is an important way to calmly face complex situations. Teachers should consider comprehensively when preparing lessons, including teaching content, student development level, and key and difficult points of teaching. In addition, it is also possible to connect with real-life situations, allowing students to experience the value of mathematics in practical applications. At the beginning of the lesson "Parabolic", the teacher can ask the question "Do you know how car headlights are designed?" From a practical perspective, it can stimulate students' interest in learning, improve their problem-solving ability, help cultivate their mathematical problem awareness, and achieve unexpected results in the classroom.

### (3) Consolidating Mathematical Foundations

The development of information technology has brought new changes to our era. Under these changes, China's education has also undergone significant reforms in order to keep up with the times. In order to cultivate students' problem awareness under the background of core literacy, the first thing to focus on is the mastery of basic mathematical knowledge. Basic mathematical knowledge is like the foundation of a big tree, only by building a solid foundation can it be further developed. During the learning process, students with weak foundations often have incomplete thinking when encountering problems, or often interrupt their thinking due to weak foundational knowledge. If students are constantly in such a situation, their self-efficacy will decrease, weakening their learning motivation and interest. And students with a solid foundation can think from multiple dimensions when discovering problems, raise questions in a short period of time, and improve learning efficiency. This requires teachers to reinforce the knowledge points they have learned in the past during teaching, stimulate students' desire to ask questions, and thus cultivate their awareness of questioning.

*Example 1.* Given  $\sin \alpha = \frac{1}{3}$ , calculate the value of  $\cos 2\alpha$ .

*Solution:* This question uses the double angle formula of cosine function for transformation. Since

$$\cos 2\alpha = 1 - 2\sin^2 \alpha,$$

so we have

$$\cos 2\alpha = \frac{7}{9}.$$

Then, by substituting the value of FF into it, the correct answer can be obtained.

In addition, in addition to the application of the double angle formula of the cosine formula, teachers can also deform the problem to consolidate the double angle formula of the sine function, and strengthen basic knowledge through training. [6] examined the development and refinement of possible mathematical models for the intellectual system of career guidance. Mathematical modeling of knowledge expression in the career guidance system, Combined method of eliminating uncertainties, Chris-Naylor method in the expert information system of career guidance, Shortliff and Buchanan model in the expert information system of career guidance and DempsterSchafer in the expert information system of career guidance method has been studied. The algorithms of the above methods have been developed. [7] discussed that according to the observations in this paper, an existing mathematical model of banking capital dynamics should be tweaked. First-order ordinary differential equations with a "predator-pray" structure make up the model, and the indicators are competitive. Numerical realisations of the model are required to account for three distinct sets of initial parameter values. It is demonstrated that a wide range of banking capital dynamics can be produced by altering the starting parameters. One of the three options is selected, and the other two are eliminated.

### (4) Creating Problem Scenarios

The new educational concept clearly proposes that in high school mathematics learning, our students need to have the ability to analyze, solve, discover, and propose problems, and form the habit of rational thinking. To achieve this requirement, we need to create problem situations. The teacher's ingenious teaching design can stimulate students to ask impulsively, allowing them to naturally think of asking "why" in the context, thereby arousing their interest in asking questions. For example, when teaching the first  $n$  items of a proportional sequence, we can create relevant situations to stimulate students' desire to explore. Firstly, ask students to fold a piece of paper multiple times and explore how many times a piece of paper can be folded at most. Then tell students, "If we consider the thickness of each piece of paper to be 0.1mm, folding it 42 times would be equivalent to the distance between the Earth and the moon. At 43 times, the thickness would exceed the distance between the Earth and the moon." This will definitely stimulate students' curiosity, and some students may also question it. At this point, the teacher can introduce the learning content of this lesson and explore this problem by studying the first  $n$  items of the proportional sequence. The learning that students engage in in such interesting situations will enable them to have a clear and profound understanding of knowledge.

(5) Encourage students to ask questions and cultivate a questioning spirit

In the classroom, students often hesitate to ask questions due to timidity, or the teacher may reject them after asking questions. Therefore, in the teaching process, teachers should encourage students to think and create opportunities for them to successfully ask questions. Regardless of whether the questions are big or small, meaningful or not, as long as they have been pondered by themselves, they can be raised, allowing students to have doubts in every corner of the classroom. Teachers should strengthen the behavior of students asking questions after they have raised them; In the classroom, teachers can also arouse students' doubts by intentionally showing mistakes. When problems arise, students are allowed to help the teacher correct them in a timely manner, allowing them to dare to question authority and boldly ask questions.

*Example 2.* Given the sum of the first  $n$  terms of the sequence  $\{a_n\}$ , find  $a_n$ .

*Solution:* The teacher can first present the incorrect solution

$$\begin{aligned} a_n &= S_n - S_{n-1} = (2^n + 1) - (2^{n-1} + 1) \\ &= 2^n - 2^{n-1} = 2^{n-1}. \end{aligned}$$

If students fail to detect the problem in a timely manner, the teacher can guide them to "Is this solution correct?" Then the students can think and identify the reason for the error. "The condition for the formula  $a_n = S_n - S_{n-1}$  to hold is  $n \geq 2$ ," and finally summarize that when using  $a_n = S_n - S_{n-1}$ , it is necessary to test the situation when  $n = 1$ , that is,

$$a_n = \begin{cases} S_1 (n=1), \\ S_n - S_{n-1} (n \geq 2, n \in N). \end{cases}$$

## V. CONCLUSION

This article explores how to cultivate high school students' awareness of mathematical problems from the perspective of core competencies. Through in-depth research, we have found that cultivating awareness of mathematical problems has profound significance in improving the mathematical abilities and learning attitudes of high school students. At the same time, some specific training methods and suggestions have also been proposed, hoping to provide effective references for high school mathematics teaching and ensure the implementation of core competencies. The cultivation of mathematical problem awareness is an important aspect of teacher teaching. With questioning, problems can be solved and the development of students' abilities in all aspects can be promoted. In the future, mathematics

teachers should deepen their practice in teaching, continue to pay attention to the cultivation of mathematical problem awareness, continuously explore and practice more effective teaching methods and means, and make greater contributions to the comprehensive development of students.

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