ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE) Vol. 6, Issue 3, March 2020

Research on case teaching of the application of Matrix elementary change method

Rui Chen[1], Liang Fang[2]

College of Mathematics and Statistics, Taishan University, Tai'an, China [1]
College of Mathematics and Statistics, Taishan University, Tai'an, China [2]

***Abstract*— The elementary transformation method of Matrix is widely used in linear Algebra. In this paper, a series of applications of the elementary transformation method are summarized based on its definition and Theorem. At the same time, some cases about the application of elementary change method are listed; we get the Implementation Scheme and teaching advantages of case-based classroom teaching.**

***Index Terms*—Matrix, Elementary Change Method, Application, Case teaching.**

I. INTRODUCTION

The elementary transformation of a matrix is a relatively abstract concept, it can be said that it is the operation of a matrix, but it is different from the familiar operation, it is not addition and multiplication, nor multiplication, but it is similar to multiplication.

A. Related definitions of elementary transformation method

Definition 1 the following three transformations are called Elementary row-column transformations of matrices:

(i)Transposition: transpose two rows (columns);

(ii) Multiplication: multiply all the elements in a row(column) by the number $k \neq 0$;

(iii) Elimination Transformation: k Times all the elements in one row (column) are added to the corresponding elements in the other row (column) [1].

Definition 2 A matrix obtained by an elementary transformation of the unit Matrix is called an elementary matrix [1].

Defining 3 A matrix with zero rows below, non zero rows above, in which the other elements of the first non-zero column of a non-zero row are all Zeros. A Matrix like this is called Row Echelon Form Matrix.

Defining 4 A matrix with zero rows below, non zero rows above, in which the first non-zero column of a non-zero row is 1and the other elements of it are all Zeros. A Matrix like this is called Row Simplest Matrix.

B. Related theorems of elementary transformation method

Theorem 1 the row elementary transformation of a matrix is equivalent to multiplying its left side by the corresponding elementary Matrix, the column elementary transformation of a matrix is equivalent to multiplying its right side by the corresponding elementary Matrix [2].

Theorem 2 A Matrix has an invertible matrix $\Leftrightarrow$ There is a finite number of elementary matrices $p_1, p_2, \cdots, p_n$, make $A = p_1 p_2 \cdots p_n$ [2].

The above proof shows that the simplest form of the row matrix of the invertible matrix is the identity matrix.

Corollary 1 A Matrix has an invertible matrix $\Leftrightarrow$ $\boldsymbol{A} \overset{r}{\sim} \boldsymbol{E}$ [1].

II. CASE TEACHING OF THE APPLICATION OF MATRIX ELEMENTARY CHANGE METHOD

A. Solving system of linear equations

The elementary transformation method can solve the non homogeneous system of linear equations and the homogeneous system of linear equations. In the process of solving the system of linear equations, the elementary transformation is the same solution transformation of the system of linear equations; we can only do elementary transformation on the augmented matrix or Coefficient Matrix of the equations. We show you two examples.

Example 1: Solving non homogeneous system of linear equations

$$\begin{cases} x_1 + x_2 + 2x_3 = 1, \\ -x_1 + x_2 + 2x_3 = -1, \\ 3x_1 + x_2 - x_3 = 6. \end{cases} \tag{1}$$

First, we write the augmented Matrix of non homogeneous system of linear equations (1),

$$B = \begin{pmatrix} 1 & 1 & 2 & 1 \\ -1 & 1 & 2 & -1 \\ 3 & 1 & -1 & 6 \end{pmatrix}.$$

Perform the elementary transformation on it, and reduce it to the simplest of lines, as follows,

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE) Vol. 6, Issue 3, March 2020

$$B \to \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

If you convert the above Matrix into a system of linear equations, you get a solution of $x_1 = 3, x_2 = 2, x_3 = -1.$

Example 2: Solving a homogeneous system of linear equations

$$\begin{cases} 2x_1 - x_2 + 3x_3 = 0, \\ -x_1 - x_2 + 2x_3 = 0, \\ 3x_1 + 5x_2 - x_3 = 0. \end{cases} \quad (2)$$

First, we write coefficient Matrix of homogeneous system of linear equations (2),

$$A = \begin{pmatrix} 2 & -1 & 3 \\ -1 & -1 & 2 \\ 3 & 5 & -1 \end{pmatrix}.$$

Perform the elementary transformation on it, and reduce it to the simplest of lines, as follows,

$$A \to \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

If you convert the above Matrix into a system of linear equations, you get a solution of $x_1 = 0, x_2 = 0, x_3 = 0.$

Through two examples, the teacher helps students realize the advantages of using the elementary variation method to solve the system of linear equations problem, which saves the trouble of solving the elimination problem with an unknown quantity, the relation between the solution of the system of linear equations and the rank of the Matrix can be further summed up, which simplifies the process of solving the system of linear equations, and can be realized by computer for system of linear equations with a large number of unknown quantities and equations [3].

B. Find the inverse matrix and solve the matrix equation

Let A is an invertible matrix of order and B *is* a $n\times s$ matrix, there are elementary matrixs : $P_1, P_2, \cdots, P_l$, make

$$A^{-1} = P_1P_2 \cdots P_l.$$

And then

$$A^{-1}A = P_1P_2 \cdots P_l A, \text{ so } E = P_1P_2 \cdots P_l A,$$
$$A^{-1}B = P_1P_2 \cdots P_l B.$$

This shows that if A is transformed into E by several elementary transformations, then the same elementary transformation of B will be transformed into $A^{-1}B$.The two forms combine to form

$$P_1P_2 \cdots P_l (A, B) = (E, A^{-1}B).$$

The meaning of the upper form:

(i) When $B=E$, we have $P_1P_2 \cdots P_l (A, E) = (E, A^{-1})$.

(ii) When A is an invertible Matrix, the solution of the equation $AX=B$ is $X=A^{-1}B$. The solution of $AX=B$ is obtained by elementary transformation of(A, B) to $(E, A^{-1}B)$, and $X=A^{-1}B$.

Example3: Find the inverse of the Matrix

$$A = \begin{pmatrix} 1 & -3 & 5 \\ -1 & 4 & -2 \\ -5 & 13 & -30 \end{pmatrix}.$$

First, we take the Matrix A and the identity Matrix, make a matrix

$$(A, E) = \begin{pmatrix} 1 & -3 & 5 & 1 & 0 & 0 \\ -1 & 4 & -2 & 0 & 1 & 0 \\ -5 & 13 & -30 & 0 & 0 & 1 \end{pmatrix}.$$

The above Matrix (A, E) is then transformed by elementary transformation to (E, X), we get

$$X = \begin{pmatrix} -94 & -25 & -14 \\ -20 & -5 & -3 \\ 7 & 2 & 1 \end{pmatrix}.$$

That is A^{-1}.

Example4: $A = \begin{pmatrix} 1 & 2 & -3 \\ 0 & -1 & 3 \\ 2 & 3 & -4 \end{pmatrix}, B = \begin{pmatrix} -3 & -1 & 2 \\ 5 & -4 & 6 \\ 6 & -2 & -1 \end{pmatrix},$

if $AX = B$, find X .

First, we take the Matrix A and the identity Matrix, make a matrix

$$(A, B) = \begin{pmatrix} 1 & 2 & -3 & -3 & -1 & 2 \\ 0 & -1 & 3 & 5 & -4 & 6 \\ 2 & 3 & -4 & 6 & -2 & -1 \end{pmatrix}.$$

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE) Vol. 6, Issue 3, March 2020

The above Matrix (A, B) is then transformed by elementary transformation to (E, X), we get

$$X = \begin{pmatrix} -56 & -45 & 113 \\ 16 & 16 & -39 \\ -7 & -4 & 11 \end{pmatrix}.$$

In introducing the definition of the Inverse Matrix, the teacher tells the students that they can solve the inverse matrix according to the defined Method of undetermined coefficients, and that they can also solve the inverse matrix through the adjoin Matrix. Through example 3, we can see that the elementary change method is simple and convenient [3].

C. Find the rank sum of a vector group and determine the Linear independence of a vector group

The rank of a vector group is equal to the rank of a matrix made up of the vector group's vectors, and the rank of a vector group linearly independent of its Necessity and sufficiency is equal to the number of vectors in the vector group, the Necessity and sufficiency of a vector group is that its rank is less than the number of vectors in the vector group. In discussing the Linear independence of a vector group, we turn the problem of a vector group Linear independence to the relation between the rank of the vector group and the number of vectors contained in the vector group, here we give an example of finding the rank sum of a vector group and determining its Linear independence [5].

Example5: Find the rank of a vector group

$$\alpha = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \beta = \begin{pmatrix} 7 \\ -5 \\ 3 \end{pmatrix}, \gamma = \begin{pmatrix} -2 \\ 6 \\ 9 \end{pmatrix}$$ and determine its linear independence.

First, we make a matrix out of a vector group

$$A = (\alpha, \beta, \gamma) = \begin{pmatrix} 1 & 7 & -2 \\ -1 & -5 & 6 \\ 1 & 3 & 9 \end{pmatrix}.$$

The Matrix is then transformed into an elementary Row echelon form,

$$A \rightarrow \begin{pmatrix} 1 & 7 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & 19 \end{pmatrix}.$$

The number of non-zero rows is 3, so $R(A) = 3$, That is, the rank of a vector group is 3.

Since the rank of a vector group is equal to the number of vectors in the vector group, it is linearly independent.

The example above shows how to use the elementary variation method to solve the rank of a vector group and to judge the Linear independence of a vector group according to the rank of the vector group. The students have a deeper understanding of rank and the validity of the elementary change method [6].

III. Conclusion

Through the discussion of the system of linear equations, it is observed in the augmented Matrix and Coefficient Matrix that these systems of linear equations are solved by a series of Matrix transformations, that is to say, the process of their transformation is the whole process of the solution. In addition to the system of equations, there are also many problems concerning the elementary transformation of matrices and its applications. In application, the study of these problems is often transformed into the study of matrices, even some problems which are completely different in nature and seem to have nothing to do with them, all of these problems can be seen as the same, which leads to the universal application of matrices in mathematics and the elementary transformation according to the relevant algorithmic rules. This paper introduces some applications of Matrix elementary transformation in Linear Algebra, and discusses the application of Matrix elementary transformation in linear Algebra by case teaching on the premise of long-term teaching experience [7].

Acknowledgment

The work is supported by Tai'an Science and Technology Development Project: Research on case teaching of linear algebra combined with learners practical ability (2019ZC251) and Teaching Reform and Research Project of Taishan University in 2018: The Theory of Probability Combining Real Life Application and Computer Simulation and the Teaching Reform of Mathematical Statistics Course.

References

[1] Department of Mathematics, Tongji University, "Linear Algebra" *Fourth Edition. Beijing: Higher Education Press*, 2003, pp.23-110.
[2] Department of Mathematics, Peking University, "Advanced Algebra" *Third Edition. Beijing: Higher Education Press*, 2003, pp. 35-180.
[3] Y.-Z. Wu, "Some applications and discussions of mat rix elementary transformation in linear algebra," *Journal of the Guangdong University of Petrochemical Technology*, 2017, 27(01), pp.71-75, 94.
[4] H.-N.Wang, "Discussion on some applications of elementary transformation of Matrix J," *Shandong industrial technology*, 2019(02), pp. 229-230.
[5] W. Xiang, "Some applications of Matrix Transformation," *Journal of Tangshan Teachers College* , 2011,33(02),pp.39-42.
[6] B.-T.He, "Some applications of elementary transformation of Matrix J," *Journal of Chongqing University of Arts and Sciences*, 2010,29(05),pp.26-28.
[7] F. Xie, "Some applications of elementary transformation of Matrix," *Journal of the Vocational School of Zhaotong Normal University*, 2004(02),PP.51-55.

Authors' Profile

ISSN (ONLINE): 2454-9762
ISSN (PRINT): 2454-9762
Available online at www.ijarmate.com

International Journal of Advanced Research in Management, Architecture, Technology and Engineering (IJARMATE) Vol. 6, Issue 3, March 2020

Rui Chen is a lecturer at Taishan University. She obtained her master's degree from Shandong University in December, 2009. Her research interests are in the areas of application of probability theory, and applied statistics in recent years. Her email id: chenruimengting@163.com

Liang Fang is a professor at Taishan University. He obtained his PhD from Shanghai Jiaotong University in June, 2010. His research interests are in the areas of cone optimization, and complementarity problems.