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The principle of small probability and its application in social life

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***Abstract*—The principle of small probability is a basic theory with strong practical application value and is widely used in social life. This article briefly analyzes the definition of small probability events, the principle of small probability, and the difference between small probability events and impossible events. The typical application of small probability principles in social life is analyzed to guide us to make scientific decisions in life. In turn, it brings a lot of convenience to life.**

***Key words*—the principle of small probability; small probability event; social life; application.**

I. THE PRINCIPLE OF SMALL PROBABILITY

A. *Meaning of the principle of small probability*

A small probability event refers to an event with a very small probability of occurrence, which is generally not greater than 0.05 or 0.01. In statistics, a small probability event is considered to be an event that cannot actually occur in an experiment. Small probability events are almost impossible to occur in one experiment, but they are almost inevitable in multiple repeated experiments. Mathematically, they are called small probability principles [1].

The principle of small probability is also called the principle of non-occurrence of small probability events. However, it should be clear that if the probability of an event A occurring in an experiment is $\varepsilon(>0)$.No matter how small, if the experiment is continuously repeated independently, then sooner or later the event A will inevitably occur once, and thus it will inevitably occur many times. Because the probability of the event not appearing in the first test is $1-\varepsilon$, the probability that none of the previous trials will occur is $(1-\varepsilon)^n$, so the probability of at least one occurrence in the previous test is $1-(1-\varepsilon)^n$. When $n\to\infty$, the probability approaches 1, this means that sooner or later there will be a probability of 1. After the appearance, take the next test as the first time, repeat the above reasoning, it is inevitable that A will appear again.

Similar situations are often encountered in daily life. For example, a person is less likely to win the lottery, but if a large number of people go to buy the lottery, there is a possibility that someone will win the lottery. The probability of a person becoming a great man is very small, but it is very possible for at least one person in millions to become a great man.

Whether small probability events can be ignored should be analyzed in detail. For example, any small probability event can be fatal to the space shuttle, and one percent of a batch of leather shoes is harmless. In the more complex problems, using the low-probability event principle can help us to dialysis the deeper background of the phenomenon of low-probability events.

B. *Inference method of small probability principle*

Theorem (Bernoulli's law of large numbers) When the number of independent repeated tests is n, the number of times of the occurrence of the event A is n_A, p is the probability of occurrence, then for any positive number ε, there is

$$\lim_{n\to\infty} P\left(\left|\frac{n_A}{n}-p\right|<\varepsilon\right)=1$$

or

$$\lim_{n\to\infty} P\left(\left|\frac{n_A}{n}-p\right|\geq\varepsilon\right)=0.$$

According to the law of large numbers, the frequency of events converges according to probability to the probability of events occurring. That is, when n is large, it is very unlikely that the frequency and probability of events will deviate greatly. If the probability of an event A occurs is very small, the principle of actual inference is used, in practical applications, when the number of tests is large, the frequency of the event A can be used instead of the probability. If the probability of an event A occurring is very small, for example 0.001, it generally occurs only once in 1,000 tests, so an event with a very small probability is almost impossible to occur in one test. In the application of probability theory, such an event is called an actual impossible event. Actual impossible events are almost impossible to occur in an experiment. This is the principle of small probability, also known as the actual impossibility principle of small

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probability events. It is the basis for the statistical hypothesis test to reject or accept the hypothesis, and it is also a very practical principle that people have summed up in long-term practice. The inference method of the small-probability principle is a counter-evidence method of probability nature. It refers to the first hypothesis, followed by the calculation based on the results of a test, and finally judged by a certain probability standard. If an unreasonable phenomenon occurs, that is, a small probability event occurs, the hypothesis is rejected. If the unreasonable phenomenon does not occur, that is, the low-probability event does not occur, the hypothesis is not rejected.

C. The difference between a small probability event and an impossible event

In long-term practice, people insist that events with very low probability are almost equivalent to impossible events in one experiment, that is, they will not occur. If a small probability event actually occurs in an experiment, people will think that the preconditions for the event have changed, or that the event is not random, but man-made, etc. This is an application of the principle of low probability. However, we should know that no matter how small the probability of a small probability event A, if the test is continuously repeated independently, then the event A will inevitably occur sooner or later, continue to repeat, and therefore will inevitably occur many times. The impossible event means that no matter how many times the test is done, the event will not happen. This shows the difference between a small probability event and an impossible event.

II. APPLICATION OF THE PRINCIPLE OF SMALL PROBABILITY

A. Quality management

The low-probability event principle is also used in the quality management theory of production and sales products. The core principle used is 3σ-Guidelines. The content is: In an experiment, a random variable X obeys a normal distribution and its value is not outside the range of $(\mu-3\sigma,\mu+3\sigma)$ [2].

Specifically: if $X \sim N(\mu,\sigma^2), Y=\dfrac{X-\mu}{\sigma} \sim N(0,1)$, then

$$P(\mu-3\sigma < X < \mu+3\sigma)$$
$$=P\left(-3<\frac{X-\mu}{\sigma}<3\right)$$
$$=\Phi(3)-\Phi(-3)$$
$$=2\Phi(3)-1$$
$$=0.9973.$$

A food factory produces a bottled snack. It is known that the quality of this bottled snack obeys the normal distribution, the quality is random variable X, the weight unit is g, that is $X \sim N(1000, 20^2)$, there is now a bottle of snack quality of 1080 g, if you check the quality of this snack, can the food processing plant pass the quality inspection?

From 3σ-Guidelines know: The probability that the quality of this bottled snack falls within the range $(940,1060)$ is close to 1. But the sample of this snack is 1080g, which is not within this range $(940,1060)$. It shows that there was a very small probability of an event in only one test, so the food processing plant could not pass the quality inspection. [3]

B. New drug testing

The principle of low-probability event has been widely used in the development of new drugs, vaccination, and drug management in the medical field.

For a special disease, new drugs have been developed. The cure rate of old drugs before the development of new drugs is known to be 0.001. Ten people now suffer from the disease, and six people have recovered after using the new drug. Has the recovery rate improved since the new drug was used [4]?

Assuming that "the rehabilitation rate of patients after the trial of new drugs has not improved", the newly developed drugs are now used by patients and 10 patients are selected. Assuming that the number of patients recovering is X, then $X \sim b(10, 0.001)$.

$$P(X \geq 6)=1-P(X<6)$$
$$=1-\sum_{k=0}^{5} C_{10}^{k} \times 0.001^{k} \times 0.999^{10-k}$$
$$\approx 0.00892.$$

The above results show that the probability that the number of patients recovering is greater than or equal to six is a small probability event, and now it occurs, indicating that this new drug is helpful for patient rehabilitation.

C. Insurance business

One company has introduced a life insurance business and now 10,000 people of the same age are insured. This business rule: For each insured person, an annual insurance premium of 200 dollars is required. If the insured person dies within one year of paying the premium, the company will compensate his beneficiary for 100,000 dollars. According to the survey data, the annual mortality rate of these people of the same age group is 0.001. Will the company lose money on this life insurance business?

Assuming that the death toll of the 10,000 insured people within one year of paying the premium is X, there is $X \sim b(10000, 0.001)$.

The company's total revenue in one year is 2000000, if $100000X > 2000000$, that is $X > 20$, the company will lose money.

10000 is a very large value, and 0.001 is very small, so it can be approximated by Poisson distribution, $\lambda = np = 10$, calculated from topic data:

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$$P(X > 20) = 1 - P(X \leq 20) \approx 1 - \sum_{k=0}^{20} \frac{10^k}{k!} e^{-10}$$
$$\approx 1 - 0.998 = 0.002.$$

From the calculation results, it can be seen that the probability that the number of insured people who die within one year of paying premiums exceeds 20 is very small. The insurance company's loss in this life insurance business is a small probability event. From here we can understand that insurance companies use the low-probability event principle to earn premiums and earn high profits.

D. *Shooting competition*

In sports schools, a student who shoots well has a shooting percentage of 70%. Now he participates in a game. In this game he shoots seven times in a row, but the number of shots is not more than two times [5]. Is the student in poor condition?

For this student, the number of times he was hit is represented by X, so $X \sim b(7, 0.7)$.

In this competition, there are the following results:

Shot 0 times:

$$P(X = 0) = 0.3^7 = 0.0002187.$$

Shot 1 time:

$$P(X = 1) = C_7^1 \times 0.7 \times 0.3^6 = 0.0035721.$$

Shot 2 times:

$$P(X = 2) = C_7^2 \times 0.7^2 \times 0.3^5 = 0.0250047.$$

The above results can be added together to obtain:

$$P(X = 0) + P(X = 1) + P(X = 2)$$
$$= 0.0287955 < 0.05.$$

From the calculation results, it can be seen that the probability of this student hitting no more than two times is indeed very small. It is a small probability event, but it happened in a competition. Therefore, it can be said that this student is in poor condition.

E. *Toy Selection*

Mary's mother went to the toy market to buy toys. The owner of the toy store said that there were 100 toys in a box, and at most 5 were bad. Mary's mother's intention was to pick out a whole box of toys and then 10 of the 100 toys in the box. She chose her own toys and found that 3 of the 10 toys were bad. In her experience, no more than two of the 10 toys picked should be bad. Mary's mother told the shopkeeper that not five of the boxes were bad. The shopkeeper objected, saying that perhaps Mary's mother had picked just three out of a box of toys. Excuse me, is Mary's mother right?

Assuming that only 5 of the 100 toys are bad, X is the number of bad toys in the 10 toys selected is the number of bad toys.

$$P(X = 3) = \frac{C_{95}^7 C_5^3}{C_{100}^{10}} \approx 0.00638,$$
$$P(X = 4) = \frac{C_{95}^6 C_5^4}{C_{100}^{10}} \approx 0.00025,$$
$$P(X = 5) = \frac{C_{95}^5 C_5^5}{C_{100}^{10}} \approx 0.000003.$$

From the above calculation results, it can be seen that the probability of extracting 10 toys, of which the number of bad toys exceeds 2 is:

$$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5)$$
$$= 0.006633 < 0.05.$$

The above results show that the probability of extracting 10 toys, of which the number of bad ones is more than 2, is very small. If a very small probability event occurs, it can be explained that Mary's mother is right.

F. *Reservation of seats*

In life, sometimes people change the conditions in which things happen, which can lead to small probability events. This application is also very useful to us. The following example is a good example:

A party was held in the school's auditorium. The first row of the auditorium had 30 seats. Now it was found that the first row had 6 empty seats adjacent to each other. The rest of the seats were filled. Is this normal?

There are 24 people sitting in 30 seats. There are C_{30}^{24} kinds of the way you sit. Now consider the six seats that are close to each other as a whole. It is equivalent to 25 seats with 24 seats and there are C_{25}^{24} kinds of the way you sit.

A means that the event "24 seats are full and 6 adjacent seats are empty", then:

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$$P(A)=\frac{C_{25}^{24}}{C_{30}^{24}}=\frac{25}{593775}.$$

From the above results, it can be seen that this is a small probability event. A small probability event occurred, indicating that the school has reserved seats in advance. This application can make small probability events happen, and it is helpful for our normal work.

G. *Statistical experiments*

Here are two famous statistical experiments that Savage, a British statistician, has visited.

A: A woman who used to drink milk said she could tell whether it was tea or milk poured into the glass first.

B: A musician claims that he can tell a piece of music from one page. He can tell the difference between Haydn and Mozart in ten experiments.

In these two statistical experiments, if the experimenter is speculating that the probability of success is 0.5 each time, then the probability of guessing ten times is $0.5^{10}=0.0009766$.

This is a small probability event that is almost impossible to occur, so the assumption is rejected. The probability that the experimenters will succeed each time is much greater than 0.5. This is not a guess, but their experience helps them. It can be seen that experience-a priori assumption is an important assumption that cannot be ignored in inference. We should use it.

Bayesian statistics is the statistical inference of basic information, general information, sample information, and transcendental information. Through the principle of small probability, transcendental information plays a very important role in statistical inference [6].

III. Conclusion

From the above discussion, we can see that the application of the principle of small probability is very extensive. It is the essence of probability theory and the basis for the survival and development of statistics. It makes people need to make analysis and judgment in the face of a large amount of data. It can make decisions based on the reasoning of specific situations, so that statistical inference has a strict mathematical theoretical basis. In fact, there are many probability problems around us. As long as we are good at grasping and using probability knowledge to solve problems, our life will be better and better [7].

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