



# Analysis of Maximum Allowable Contact Pressure and Von Mises Stresses in Spur Gear

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**Abstract**— FEA model could be utilized to mimic contact between two bodies precisely by confirmation of contact burdens between two good gears in contact. These outcomes uncover that Maximum reasonable Contact Pressure and Von Mises Stresses on involute pair of good apparatus teeth. Both systematic and ANSYS results take after same pattern. Toward the end of the contact, the anxiety expanded abruptly to a high esteem near the greatest worth, at this stage a sliding was happened in the contact district at the most extreme anxiety focuses.

**Index Terms**—FEA Model, Maximum reasonable Contact Pressure, Von Mises Stresses, Good Gears.

## I. INTRODUCTION

The expanding interest for calm force transmission in machines, vehicles, lifts and generators, has made a developing interest for a more exact examination of the qualities of apparatus frameworks. In the car business, the biggest producer of apparatuses, higher unwavering quality and lighter weight riggings are important as lighter vehicles keep on being sought after. What's more, the accomplishment in motor commotion lessening advances the generation of calmer apparatus sets for further clamor diminishment. Clamor lessening in rigging sets is particularly basic in the quickly developing field of office-robotization hardware as the workplace environment is unfavorably influenced by commotion, and machines are assuming a regularly augmenting part in that environment. At last, the main powerful approach to accomplish gear clamor lessening is to diminish the vibration connected with them. The decrease of commotion through vibration control must be accomplished through exploration endeavors by experts in the field. Be that as it may, a lack of these masters exists in the more up to date, lightweight commercial ventures in Japan for the most part on the grounds that less youngsters are gaining practical experience in rigging innovation today and customarily the authorities utilized in overwhelming businesses tend to stay where they are.

. In ANSYS, one can click File > Import > IGES > and check No defeaturing and Merge coincident key points.

(i) Pitch circle - It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

(ii) Pitch circle diameter - It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as pitch diameter.

(iii) Pitch point - It is a common point of contact between two pitch circles.

(iv) Pitch surface - It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

(v) Pressure angle or angle of obliquity - It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by  $\phi$ . The standard pressure angles are  $14\frac{1}{2}^\circ$  and  $20^\circ$ .

(vi) Addendum - It is the radial distance of a tooth from the pitch circle to the top of the tooth.

(vii) Dedendum - It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

(viii) Addendum circle - It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

(ix) Dedendum circle - It is the circle drawn through the bottom of the teeth. It is also called root circle.

(x) Circular pitch - It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by  $P_c$ .

$$P_c = \frac{\pi D}{T}$$

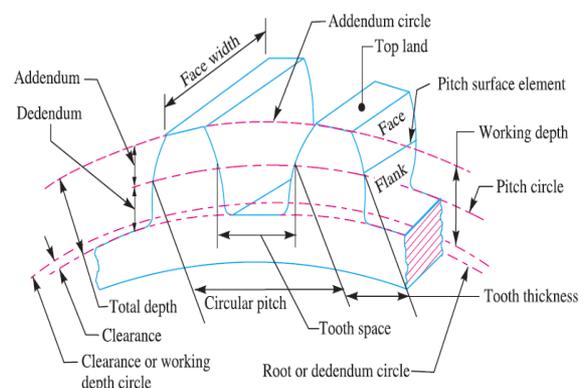


Fig.1. Terms used in gears.



(xi) Module - It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by  $m$ . Mathematically, Module,  $m = D / T$

(xii) Backlash - It is the difference between the tooth space and the tooth thickness, as measured on the pitch circle.

(xiii) Total depth - It is the radial distance between the addendum and the dedendum circle of a gear. It is equal to the sum of the addendum and dedendum.

(xiv) Working depth - It is radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

(xv) Tooth thickness - It is the width of the tooth measured along the pitch circle.

(xvi) Tooth space - It is the width of space between the two adjacent teeth measured along the pitch circle.

(xvii) Face width - It is the width of the gear tooth measured parallel to its axis.

(xviii) Profile - It is the curve formed by the face and flank of the tooth.

(xix) Fillet radius - It is the radius that connects the root circle to the profile of the tooth.

(xx) Path of contact - It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

(xxi) Length of the path of contact - It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

#### A. ADVANTAGES OF GEAR DRIVES

- It transmits exact velocity ratio.
- It may be used to transmit large power.
- It may be used for small centre distances of shafts.
- It has high efficiency.

#### B. DISADVANTAGES OF GEAR DRIVES

- The manufacture of gears require special tools and equipment, therefore it is costlier than other drives.
- The error in cutting teeth may cause vibrations and noise during operation.
- It requires suitable lubricant and reliable method of applying it, for the proper operation of gear drives.

[1] investigated a spur gear set using FEM. The contact stresses were examined using a two dimensional FEM model. The bending stress analysis was performed on different thin rimmed gears. The contact stress and bending stress comparisons were given in his studies. Based on this survey, gears are made up of steel usually heat treated in order to combine properly the toughness and tooth hardness. The variations of the whole gear body stiffness arising from the gear body rotation due to bending deflection, shearing displacement and contact deformation.

## II. PROBLEM SPECIFICATION

The problem chosen for Spur Gear drive used in Rock Crusher which transmits Power ( $P$ ) = 30 kW Speed ( $n$ ) = 1200 rpm; Gear Ratio,  $i = 3$ ; Pressure Angle,  $\alpha = 20^\circ$  Full Depth Involute Tooth Profile. It has medium shock conditions. The gear is to work 8 hours per day and for 3 years.

#### A. DESIGN PROCEDURE OF A SPUR GEAR DRIVE

##### STEP: 1

Selection of Material. From PSG DDB Pg. No. 8.4, based on speed ratio,  $i < 4$ , The Material chosen is C 45 Steel. Assume Pinion and Gear is made up of same material.

##### STEP: 2

Calculate Life cycle

Life in cycles = (Life in hours) x 60 x Rpm

$$= (20,000) \times 60 \times 1200 \quad [\text{Assume} = 20,000 \text{ Hours}]$$

Life in cycles =  $1.44 \times 10^9$  Cycles.

##### STEP: 3

Calculate Young's Modulus,  $E$ , Allowable Bending Stress,  $[\sigma_b]$  and

Allowable Surface stress,  $[\sigma_c]$

From PSG DDB Pg. No. 8.14 (Table 9)

Steel, Young's Modulus,  $E = 2.15 \times 10^6 \text{ Kg/cm}^2 = 2.15 \times 10^5 \text{ N/mm}^2$ .

Allowable Bending Stress,  $[\sigma_b]$  - From PSG DDB Pg. No. 8.18

$$[\sigma_b] = \frac{1.4K_{bl}\sigma_{-1}}{nK_\sigma}$$

$K_{bl}$  = Life factor for Bending ---- From PSG DDB Pg. No. 8.20 (Table 22) ---

For Steel Surface Hardness  $HB > 350$ , Life Cycle  $\geq 25 \times 10^7$   $K_{bl} = 0.7$

$n$  = Factor of Safety --- From PSG DDB Pg. No. 8.19 (Table 20) ---- Steel --Tempered --- $n=2.0$

$K_\sigma$  = Stress Concentration factor ----- From PSG DDB Pg. No. 8.19 (Table 21)

Steel, Normalised,  $0 \leq X \leq 0.1$  ;  $K_\sigma = 1.5$

$\sigma_{-1}$  = Endurance Limit Stress, From PSG DDB Pg. No. 8.19 (Table 19)

$$\sigma_{-1} = 0.35 \sigma_u + 1200 \text{ (Kg/cm}^2\text{)}$$



$$\sigma_{-1} = 0.35 \sigma_u + 120 \text{ (N/mm}^2\text{)}$$

$\sigma_u \geq 63 \text{ Kgf/mm}^2$ . [From PSG DDB Pg. No. 8.5 (Table 7)] ----

$$\sigma_u \geq 630 \text{ N/mm}^2.$$

$$\sigma_{-1} = (0.35 \times 630) + 120$$

$$\sigma_{-1} = 340.5 \text{ N/mm}^2.$$

$$[\sigma_b] = \frac{1.4 * 0.7 * 340.5}{2 * 1.5}$$

$$[\sigma_b] = 111.23 \text{ N/mm}^2.$$

Allowable Design Surface Stress,  $[\sigma_c]$  --- From PSG DDB Pg. No. 8.16

$$[\sigma_c] = C_B HB k_{cl}$$

$k_{cl}$  = Life Factor, From PSG DDB Pg. No. 8.17 (Table 17)

$$\text{Steel} > 350 \geq 25 \times 10^7, k_{cl} = 0.585$$

$C_B$  --- Hardened/ Tempered --- PSG DDB Pg. No. 8.16, (Table 16),  $C_B = 26.5$

HB --- Surface Hardness --- PSG DDB Pg. No. 8.16, (Table 16), HB = 40 to 55

$$HB = 55$$

$$[\sigma_c] = 26.5 * 55 * 0.585$$

$$[\sigma_c] = 852.64 \text{ N/mm}^2.$$

#### STEP: 4

Calculate Initial Design Twisting Moment,  $[M_t]$ ,

From PSG DDB Pg. No. 8.15,  $[M_t] = k k_d M_t$

Take,  $k k_d = 1.3$  (Symmetric – PSG DDB Pg. No. 8.15)

$$M_t = \frac{60 * P}{2\pi n_1}$$

$$= \frac{60 * 30 * 10^3}{2 * \pi * 1200}$$

$$M_t = 238.73 \text{ N-m}$$

$$M_t = 238.73 \times 10^3 \text{ N-mm}$$

$$[M_t] = 1.3 * 238.73 * 10^3$$

$$[M_t] = 310.34 \times 10^3 \text{ N-mm}.$$

#### STEP: 5

Calculate Centre Distance,  $a$  --- From PSG DDB Pg. No. 8.13, For SPUR,

$$a \geq (i \pm 1) \sqrt[3]{\left[\left(\frac{0.74}{[\sigma_c]}\right)^2 * \frac{E[M_t]}{i\psi}\right]} \quad (\text{Assume,}$$

$\psi = 0.3$ , For SPUR )

$$a \geq (3+1) \sqrt[3]{\left[\left(\frac{0.74}{[85264]}\right)^2 * \frac{2.15 * 10^5 * [31034 * 10^3]}{3 * 0.3}\right]}$$

$$a \geq 152.89 \text{ mm}$$

Centre Distance,  $a=155 \text{ mm}$ .

#### STEP: 6

Calculate Number of Teeth  $Z_1$  and  $Z_2$ ,

Assume,  $Z_1 = 18$  ( $\alpha = 20^\circ$  Full Depth Involute Profile)

From PSG DDB Pg. No. 8.1,  $i = \frac{Z_2}{Z_1}$

$$Z_2 = i * Z_1 = (3 * 18)$$

$$Z_2 = 54$$

#### STEP: 7

Calculate Module,  $m$  --- From PSG DDB Pg. No. 8.22,

$$m = \frac{2a}{Z_1 + Z_2}$$

$$= \frac{2 * 155}{18 + 54}$$

$$m = 4.03 \text{ mm}$$

Recommended Series of Module -  $m$ , From PSG DDB Pg. No. 8.2 (Table 1)

Module,  $m = 5 \text{ mm}$

#### STEP: 8

Re-calculate centre distance,  $a = \frac{m(Z_1 + Z_2)}{2}$

$$= \frac{5 * (18 + 54)}{2}$$

$$a = 180 \text{ mm}$$

#### STEP: 9

Re-calculate Twisting Moment,  $[M_t]$  --- From PSG DDB Pg. No. 8.15,  $[M_t] = k k_d M_t$

Face Width,  $b = \psi * a$  (From PSG DDB Pg. No. 8.14 --- Table 10)

$$= (0.3 * 180)$$

Face Width,  $b=54 \text{ mm}$



$$\text{Pitch Diameter of Pinion, } d_1 = m * Z_1 \\ = (5 * 18)$$

$$\text{Pitch Diameter of Pinion, } d_1 = 90 \text{ mm}$$

$$\text{Pitch Line Velocity, } v = \frac{\pi d_1 n_1}{60 * 1000} \\ = \frac{\pi * 90 * 1200}{60 * 1000}$$

$$\text{Pitch Line Velocity, } v = 5.34 \text{ m/s}$$

K = Load Concentration Factor --- From PSG DDB Pg. No.

$$8.15 \text{ (Table 14) --- } \psi_p = \frac{b}{d_1} \\ = (54/90) \\ \psi_p = 0.6$$

Based on  $\psi_p$ , From PSG DDB Pg. No. 8.15, (Table 14), For,  $\psi_p = 0.6$ ,

Bearings close to gears and symmetrical, K = 1.03

$K_d$  = Dynamic Load Factor --- From PSG DDB Pg. No. 8.16, (Table 15)

Based on pitch line velocity,  $v = 5.34$  m/s, IS quality 8, HB > 350,  $K_d = 1.4$

$$[M_t] = 1.03 * 1.4 * 238.73 * 10^3$$

$$M_t = 344.24 \times 10^3 \text{ N-mm.}$$

#### STEP: 10

Check the Stress  $[\sigma_b]$  &  $[\sigma_c]$

Bending Stress,  $\sigma_b$  --- From PSG DDB Pg. No. 8.13A,

$$\sigma_b = \frac{i \pm 1}{amby} [M_t] \leq [\sigma_b]$$

y- From PSG DDB Pg. No. 8.18, (Table 18) ---- Based on  $Z_1 = 18$  teeth,

Addendum Modification Coefficient, X = 0,  $y = 0.377$

$$\sigma_b = \frac{3+1}{180 * 5 * 54 * 0.377} [344.24 * 10^3]$$

$$= 75.15 \text{ N/mm}^2 < [\sigma_b], \text{ Hence Design is Safe.}$$

Contact Stress,  $\sigma_c$  --- From PSG DDB Pg. No. 8.13,

$$\sigma_c = 0.74 \frac{i \pm 1}{a} \sqrt{\frac{i \pm 1}{ib} E [M_t]} \leq [\sigma_c]$$

$$= 0.74 * \frac{3+1}{180} \sqrt{\frac{3+1}{3 * 54} * 2.15 * 10^5 * [344.24 * 10^3]}$$

$$= 702.97 \text{ N/mm}^2 < [\sigma_c], \text{ Hence Design is Safe}$$

#### STEP: 11

Calculate Forces acting on Gear, From PSG DDB Pg. NO. 8.50 & 8.51 (Lewis Eqn.)

(i) Tangential Force ( $F_t$ ):

$$F_t = \frac{P}{v} K_o \quad (\text{Assume, } K_o = 1.25 \text{ ---- Spur Gear})$$

$$F_t = \frac{30 * 10^3}{5.34} * 1.25$$

$$F_t = 7022.47 \text{ N}$$

(ii) Initial Dynamic Load ( $F_d$ ):

$$F_d = \frac{F_t}{C_v}, \quad C_v \text{ --- From PSG DDB Pg. No. 8.51,}$$

$$C_v = \frac{(3 + V_m)}{3}, \quad v_m = \text{Pitch line velocity,}$$

$$C_v = \frac{(3 + 5.34)}{3} = 2.78,$$

$$F_d = \frac{7022.47}{2.78}$$

$$F_d = 2526.06 \text{ N}$$

(iii) Beam Strength ( $F_s$ ):

$$F_s = \pi m b [\sigma_b] y$$

$$F_s = \pi * 5 * 54 * 111.23 * 0.377$$

$$F_s = 35569.43 \text{ N}$$

(iv) Dynamic Load ( $F_d$ ):

$$F_d = F_t + \left[ \frac{0.164 v_m (cb + F_t)}{0.164 v_m + 1.485 \sqrt{cb + F_t}} \right]$$

c---- From PSG DDB Pg. No. 8.53, (Table 41)

For 20° Full Depth (Steel and Steel),  $c = 11860$  e

e--- From PSG DDB Pg. No. 8.53, (Table 42)

For Module,  $m = 5$  mm,  $e = 0.025$ , (carefully cut gears)

$$c = 11860 * 0.025$$

$$c = 296.5 \text{ mm}$$



$$F_d = 7022.47 + \left[ \frac{0.164 * 5.34 * 10^3 * ((296.5 * 54) + 7022.47)}{0.164 * 5.34 * 10^3 + 1.485 \sqrt{(296.5 * 54) + 7022.47}} \right]$$

$$F_d = 25341.64 \text{ N}$$

$F_d < F_s$ , It means the gear tooth has adequate beam strength and it will not fail by breakage. Thus *Design is satisfactory*.

(v)Wear Load( $F_w$ ):

$$F_w = d_1 Q k b$$

$$Q = \frac{2i}{i+1} = \frac{2 * 3}{3+1}$$

$$Q = 1.5$$

$$k = \frac{[\sigma_c^2] \sin \alpha \left[ \frac{1}{E_1} + \frac{1}{E_2} \right]}{1.4}$$

$$k = \frac{[(852.64)^2] \sin 20 \left[ \frac{1}{2.15 * 10^5} + \frac{1}{2.15 * 10^5} \right]}{1.4}$$

$$k = 1.652$$

$$F_w = 90 * 1.5 * 1.652 * 54$$

$$F_w = 12043.08 \text{ N}$$

Initial Dynamic Load,  $F_d < F_w$ , It means the gear tooth has adequate wear capacity and it will not wear out. Thus *Design is satisfactory*. For checking wear load,  $F_w$ . Check the values with Initial Dynamic load,  $F_d$

**STEP: 12**

Calculate Basic Dimensions of Pinion and Gear ---

PSG DDB Pg. No. 8.22, (Table 26)

(i)Module,  $m = 5 \text{ mm}$

(ii)Face Width,  $b = 54 \text{ mm}$

(iii)Height factor,  $f_o = 1$  (Full Depth)

(iv)Bottom Clearance,  $c = 0.25 \text{ m} = (0.25 * 5)$ ,  $c = 1.25 \text{ mm}$

(v)Tooth Depth,  $h = 2.25 \text{ m} = (2.25 * 5)$ ,  $h = 11.25 \text{ mm}$

(vi)Pitch Circle Diameter,  $d_1 = m Z_1 = (5 * 18)$ ,

$$d_1 = 90 \text{ mm}$$

$$d_2 = m Z_2 = (5 * 54),$$

$$d_2 = 270 \text{ mm}$$

$$\text{Diameter, } d_{a1} = (Z_1 + 2f_o) m = [18 + (2 * 1)] * 5,$$

$$d_{a1} = 100 \text{ mm}$$

$$d_{a2} = (Z_2 + 2f_o) m = [54 + (2 * 1)] * 5,$$

$$d_{a2} = 280 \text{ mm}$$

$$\text{(viii)Root Diameter, } d_{f1} = (Z_1 - 2f_o) m - 2c = [18 - (2 * 1)] * 5 - (2 * 1.25),$$

$$d_{f1} = 77.5 \text{ mm}$$

$$d_{f2} = (Z_2 - 2f_o) m - 2c = [54 - (2 * 1)] * 5 - (2 * 1.25),$$

$$d_{f2} = 257.5 \text{ mm}$$

Hence both Gears are made up of same material.

Design of pinion alone is sufficient. So no need to check for gear

The formula given by Lewis equation still serves as the basis for gear-tooth bending stress analysis. Fig.2. shows the tangential force for the spur gear is 7023 N

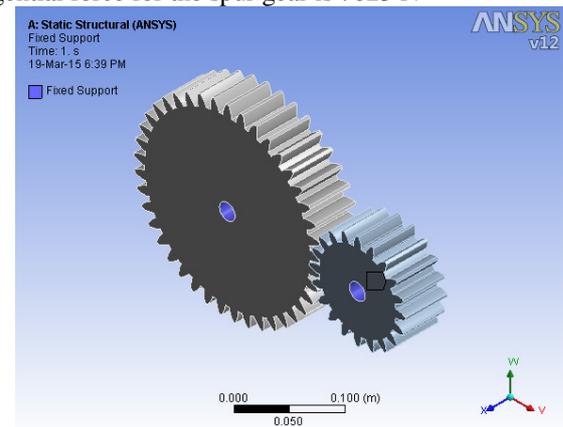


Fig.2. Loading condition

### III. RESULTS AND DISCUSSION

Fig.3. shows the Von Mises stresses on the root of tooth were carried out in order to know if they match the results from ANSYS.

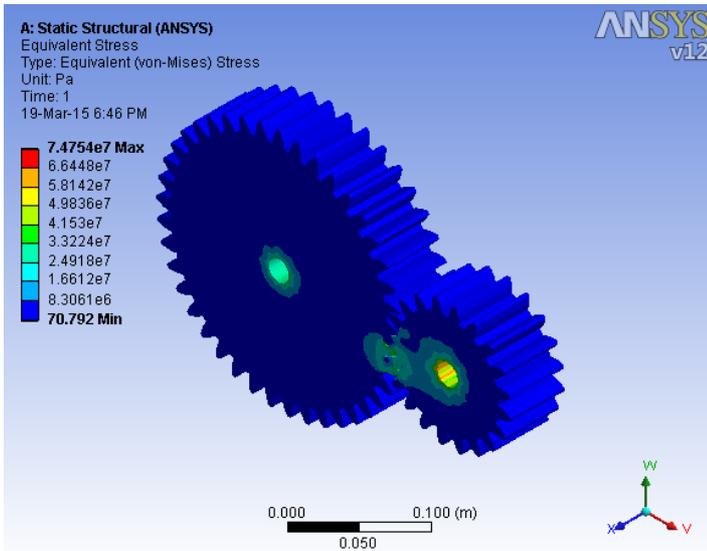


Fig.3. Von Mises stresses

The stress analysis has been done by ANSYS software, where the results have been presented by contours and numerical values. The maximum Von Mises stress value for this model was 74 MPa. Figure 5.3 shows the zoom model of the Von Mises stresses in spur gear model.

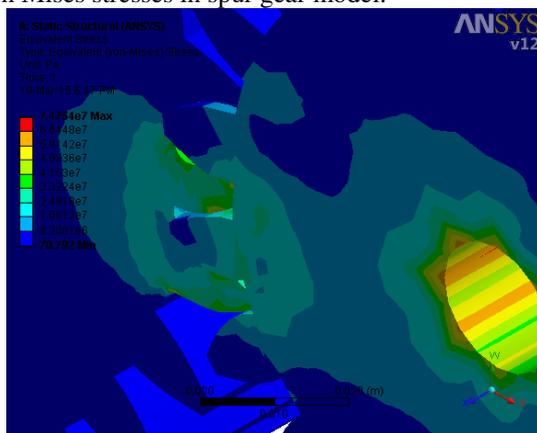


Fig.4. Zoom model of spur gear

TABLE I  
COMPARISON OF RESULTS

Parameters	Analytical Values	ANSYS Values	% of error
Contact Pressure (MPa)	702.97	699.59	0.48
Von Mises Stress (MPa)	75.15	74.75	0.53

Table I shows the comparison of Analytical and ANSYS values.

Fig.5. show the relationship form these stress results along with the prorogation of contact point location.

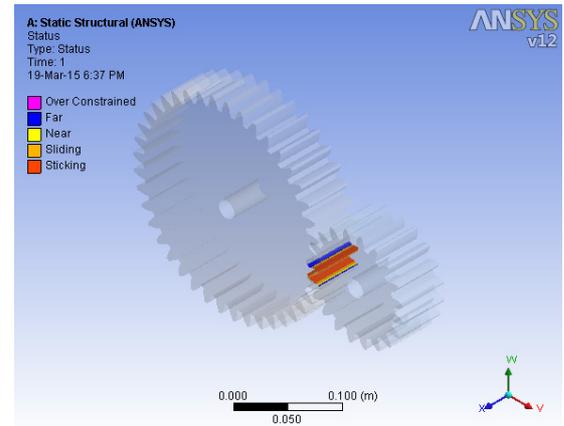


Fig.5. Contact Region Status

#### IV. CONCLUSION

FEA Model could be utilized to mimic contact between two bodies precisely by confirmation of contact burdens between two goad gears in contact. These outcomes uncover that Maximum reasonable Contact Pressure and Von Mises Stresses on involute pair of goad apparatus teeth. Both systematic and ANSYS results take after same pattern. Toward the end of the contact, the anxiety expanded abruptly to a high esteem near the greatest worth, at this stage a sliding was happened in the contact district at the most extreme anxiety focuses.

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