

# Nomogram-based Synthesis of Complex Planar Mechanisms, Part III: Six Bar-Three Sliders Mechanism (Design I)

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**Abstract**— This paper presents the synthesis of a six bar – three sliders planar mechanism for motion generation without need to sophisticated procedures like solution of nonlinear equations or any optimization technique. The idea is using a nomogram-based synthesis technique invented by the author to help mechanical engineers to synthesize complex mechanisms without need to any computer applications. The new simple synthesis technique is capable of synthesizing the 6bar – 3 sliders mechanism with time ratio up to 3 and normalized stroke up to 8 with transmission angle within the recommended range. A case study is presented to prove the efficiency and accuracy of the suggested technique.

**Index Terms**— Nomogram-based synthesis, 6 bar-3 sliders mechanism (design I), mechanism synthesis .

## I. INTRODUCTION

This is the third application on using nomogram-based synthesis invented by the author. The mechanism in hand is one of the complex mechanisms capable of generation motion with high time ratio and high normalized stroke.

Hodges and Pisano (1991) presented a kinematic analysis of constant velocity, straight line coupler-point motion of two planar mechanisms. One of them was the 5R2P Stephenson 6 bar linkage. The second one was the 7R Watt 6 bar linkage. They showed that to maintain acceptable transmission angle requirements, the velocity error and scan fraction requirements may be as little as 2 % as much as 35 % respectively [1]. Hongying, Zhixing, Dewei and Junsheng (2003) studied a numerical comparison method of planar 6-bar dwell mechanism synthesis. They illustrated their method by computer simulation on mechanism kinematics [2]. Shiakolas, Koladiya and Kebrle (2005) presented a methodology combining differential evolution, an evolutionary optimization scheme and the geometric centroid of precision positions technique for mechanism synthesis. They synthesized a six bar mechanism for dwell and dual dwell with prescribed timing and transmission angle constraints [3]. Ridley (2006) described a teaching exercise demonstrating the design and analysis of a six bar mechanism used to pour liquid from a bottle. He presented the teaching method along with typical outcomes of the design project [4].

Cruz et. al. (2010) presented a genetic algorithm based

approach for optimization of the kinematic synthesis of a six bar Watt type mechanism for prosthetic knee applications. The designed trajectory corresponded to the trajectory of the knee during a normal gait cycle [5]. Pira, Bajraktari, Conatu and Ymeri (2011) described possible ways in which Burmester theory can be utilized in the synthesis of six bar planar mechanism prescribing four or five precision points [6]. Bulatovic and Dordevic (2012) presented the optimal dimensional synthesis of a six bar linkage with rotational constraints in which a point on the second dyad generates the desired path. The aim of their synthesis was to bring the generated path as close to the given path as possible [7].

Wang (2013) established kinematics mode of the six bar drawing mechanism by bar group method and produced simulated system by Visual Basic. Optimization results showed that the mechanism kinematics was improved greatly [8]. Ajay, Shetty, Spandana and Seetharamu (2014) studied the analysis a six bar mechanism used in a precision deep drawing press. They used genetic algorithm for optimizing the dimensions of the mechanism corresponding to a chosen objective function [9]. Buskiewicz (2014) employed a technique for path synthesis for design of a feeder for carrying products between two points. The feeder had one DOF linkage of six links connected by revolute joints. He presented the mathematical basis of the synthesis concept [10].

Hassaan (2015) proposed a nomogram-based synthesis technique for planar complex mechanisms without need to solution of nonlinear equations or the application of optimization techniques. He applied his new synthesis technique to the synthesis of a six bar – two sliders mechanism [11] and to the synthesis of a six bar – one slider mechanism [12].

## II. MECHANISM

The planar 6 bar – 3 sliders mechanism under study is shown in Fig.1. It is a simplified mechanism of the quick return motion mechanism invented by the Arabic mechanical engineer Ibn Ismail Al-Jazari in the beginning of the 13<sup>th</sup> century AC [13]. The construction and operation of this mechanism is presented before by the author [14].

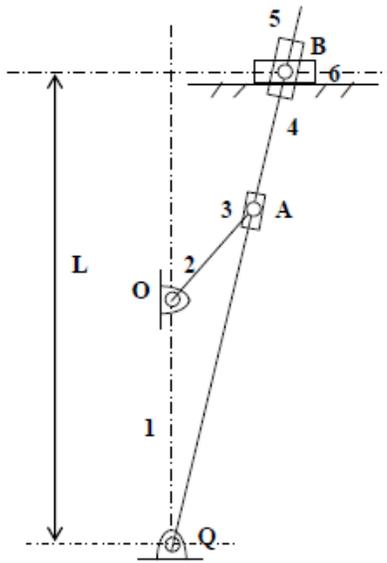


Fig.1 The 6-bar 3-sliders planar mechanism (design I) [14].

Here, I call it design 1 because simply the author is intended to synthesize another structures of the 6 bar – 3 sliders mechanism where he will designate them as design 2, design 3 and so on.

### III. PERFORMANCE PARAMETERS

The performance parameters of the mechanism are: mechanism stroke, time ration, minimum transmission angle and maximum transmission angle. To derive those functional parameters, the mechanism is drawn in its two limiting positions as shown in Fig.2.

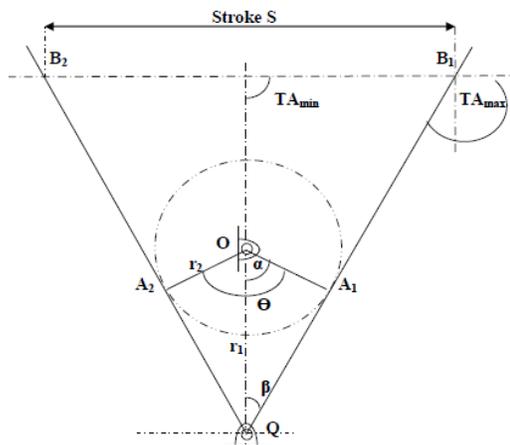


Fig.2 Limiting positions of the mechanism.

The limiting positions of the output slider are B<sub>1</sub> and B<sub>2</sub>. Corresponding to it slider 3 positions A<sub>1</sub> and A<sub>2</sub> respectively. The mechanism stroke is S (B<sub>1</sub>B<sub>2</sub>). The crank angle corresponding to the return stroke is  $\square$ . The maximum transmission angle is TA<sub>max</sub> at the limiting position of the oscillating lever. The minimum transmission

angle is TA<sub>min</sub> when the oscillating lever is in the vertical position (making 90 degrees with slider 6 centerline).

The mathematical models of the functional parameters of the mechanism in Fig.1 are as follows:

- Time ratio, TR:

The time ratio of a mechanism is defined as the ratio between the time of the forward stroke to the time of the return stroke. In terms of the crank angle, it is defined for a constant speed crank by (see Fig.2) :

$$TR = (360 - \square) / \square \quad (1)$$

Where:

$$\square = 2\alpha \quad (2)$$

$$\text{And } \alpha = \cos^{-1}(r_2/r_1) \quad (3)$$

Using normalized dimensions by referring all the dimensions to the crank length r<sub>2</sub>. The normalized length of the frame, r<sub>1n</sub> becomes:

$$r_{1n} = r_1/r_2 \quad (4)$$

Combining Eqs.3 and 4 gives the angle  $\alpha$  as:

$$\alpha = \cos^{-1}(1/r_{1n}) \quad (5)$$

- Stroke, S:

The mechanism stroke S using the trigonometric relations of the triangles in Fig.2 is given by:

$$S = 2L \tan \beta \quad (6)$$

Where:

$$\beta = 90 - \alpha \quad (7)$$

Dividing Eq.6 by r<sub>2</sub> provides the normal stroke S<sub>n</sub> in terms of the normalized length L<sub>n</sub> (L/r<sub>2</sub>). That is:

$$S_n = 2L_n \tan \beta \quad (8)$$

- Maximum transmission angle, TA<sub>max</sub>:

Using the geometry of Fig.2, the maximum transmission angle, TA<sub>max</sub> is given by:

$$TA_{max} = 90 + \beta \text{ degrees} \quad (9)$$

And the minimum transmission angle, TA<sub>min</sub> is given by:

$$TA_{min} = 90 \text{ degrees} \quad (10)$$

### IV. KINEMATIC FUNCTIONS OF THE MECHANISM

1. Mechanism dimensionless parameters: The mechanism under study has only two normalized dimensions r<sub>1n</sub> and L<sub>n</sub>, where r<sub>1n</sub> is the ground normalized dimension (r<sub>1</sub>/r<sub>2</sub>) and L<sub>n</sub> is another ground dimension from the fixed origin Q to the line of motion of the output slider (L<sub>n</sub> = L/r<sub>2</sub>) and r<sub>2</sub> is the crank OA length.
2. Ranges of the normalized dimensions of the mechanism: This mechanism was synthesized by the author for optimal normalized dimensions leading to maximum time ratio [14]. The normalized dimensions revealed was:

$$1.40 \leq r_{1n} \leq 2.76$$

$$2.58 \leq L_n \leq 4.00$$

3. Time ratio: That is the time ratio given by Eq.1. It

is function only of the normalized ground  $r_{1n}$ . The effect of  $r_{1n}$  on the time ratio TR for a range from 1.4 to 2.8 is shown graphically in Fig.3.

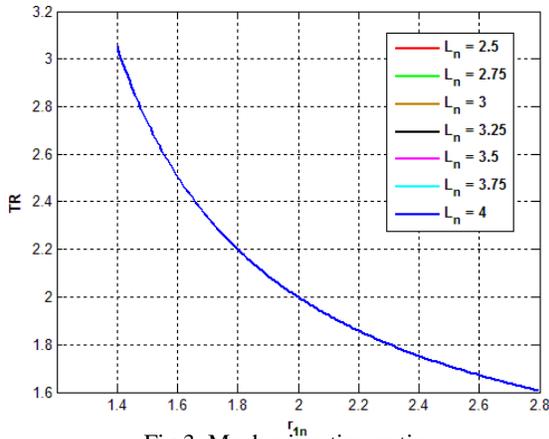


Fig.3. Mechanism time ratio.

4. Mechanism stroke: The mechanism normalized stroke is given by Eq.8. It is function of both the normalized ground dimensions  $r_{1n}$  and  $L_n$ . The effect of both parameters on the normalized stroke is shown graphically in Fig.4.

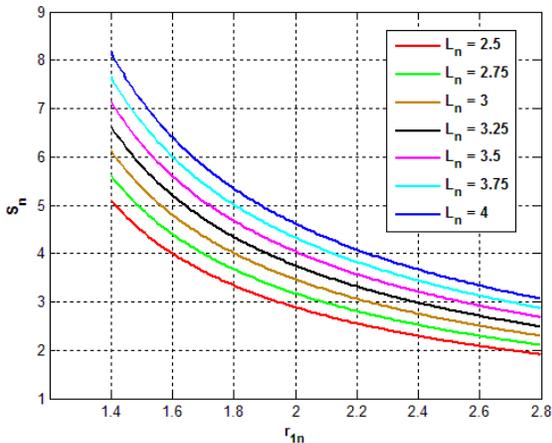


Fig.4. Mechanism normalized stroke.

5. Mechanism maximum transmission angle: The maximum transmission angle,  $TR_{max}$  is given by Eq.9 which is function only of the ground normalized dimension  $r_{1n}$ . It is shown graphically in Fig.5. Most values are within the recommended range for the mechanism transmission angle ( $45 \leq TA \leq 135$  degrees) [15].

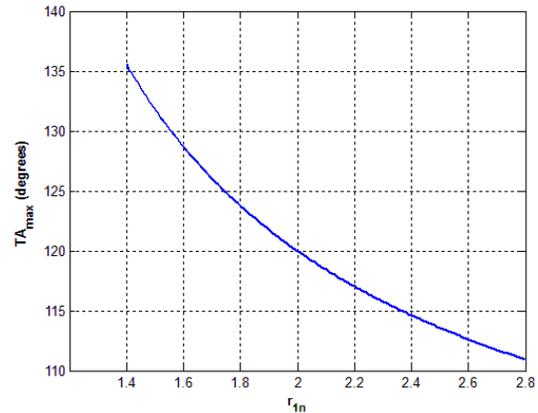


Fig.5. Mechanism maximum transmission angle.

6. Minimum transmission angle: The minimum transmission angle is independent on both  $r_{1n}$  and  $L_n$  and has a constant value of 90 degrees according to Eq.10. This independence is presented graphically in Fig.6 for purpose of constructing the synthesis nomogram.

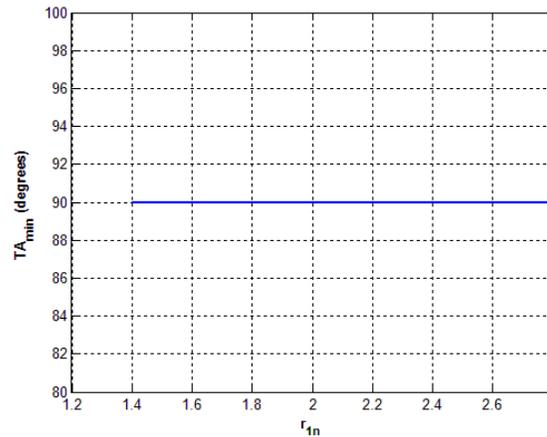


Fig.6. Mechanism minimum transmission angle.

### V. NOMOGRAM-BASED SYNTHESIS

A nomogram was suggested by the author to be used as the basis of what is he called ‘Nomogram-based synthesis’ [11], [12]. The nomogram consists of the four performance functions of the mechanism under study which are: its time ratio, its normalized stroke, its minimum transmission angle and its maximum transmission angle. The nomogram is shown in Fig.7 compiled from Figs.3, 4, 5 and 6.

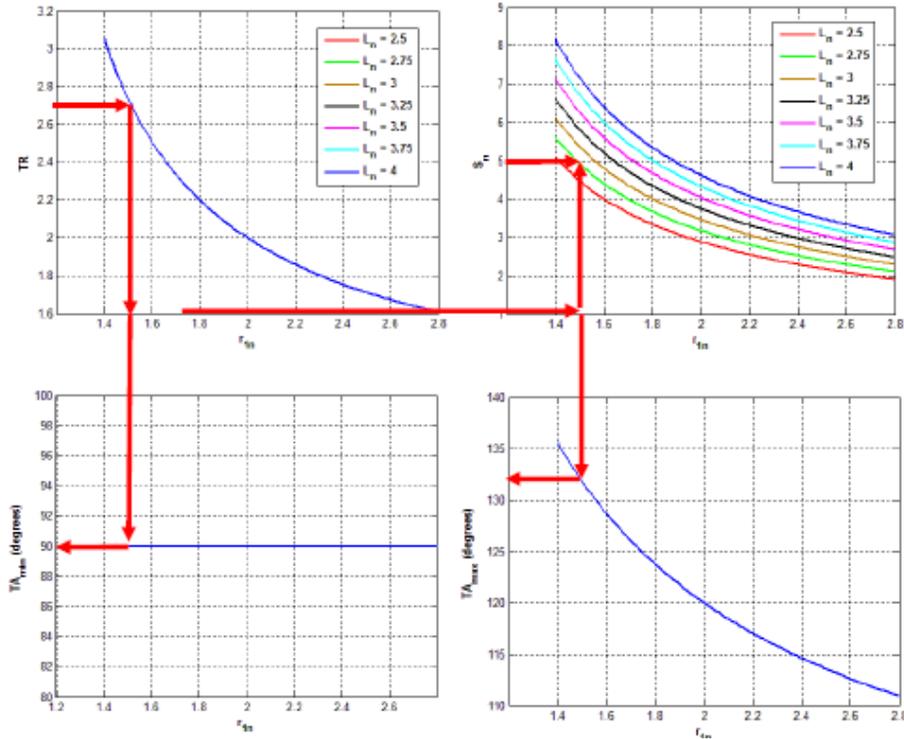


Fig.7 Synthesis nomogram for the 6 bar – 3 sliders mechanism

## VI. NOMOGRAM-BASED SYNTHESIS

The nomogram is used in the mechanism synthesis as follows (see Fig.7):

1. Suppose that it is required to synthesize a 6 bar – 3 sliders mechanism of the line diagram shown in Fig.1 such that it has a time ratio of 2.7 and a normalized stroke of 5.
2. In the nomogram of Fig.7, draw a horizontal line in the TR graph at TR = 2.7 to intersect with the curve in the corresponding value of  $r_{1n}$  as 1.5.
3. Move with  $r_{1n}$  (1.5) to the  $S_n$  graph and draw a vertical line. Draw another horizontal line at the desired normalized stroke of 5. The intersection of the two lines assigns the value of  $L_n$  as  $L_n = 2.75$ .
4. Moving down with the drawn vertical line in the TA graph to the  $TA_{min}$  graph locates the minimum transmission angle as 90 degrees.
5. Moving down with the drawn vertical line in the  $S_n$  graph to the  $TA_{max}$  graph locates the maximum transmission angle as 132 degrees.

## VII. OUTPUT MOTION OF THE SYNTHESIZED MECHANISM

The application of the nomogram-based synthesis technique to a 6 bar 3 sliders mechanism has led to the normalized dimensions (as a case study):

$$r_{1n} = 1.5$$

$$L_n = 2.75$$

The analytical kinematic functions of the mechanism using the mechanism parameters are as follows:

- Time ratio: TR = 2.7352 with -1.3 % error.
- Normalized stroke:  $S_n = 4.9193$  with 1.64 % error.
- Minimum transmission angle: 90 degrees.
- Maximum transmission angle:  $TA_{max} = 131.8103$  with 0.14 % difference from nomogram value.

The displacement diagram of the slider output using the synthesis parameters  $r_{1n}$  and  $L_n$  for one revolution of the mechanism crank is shown in Fig.8.

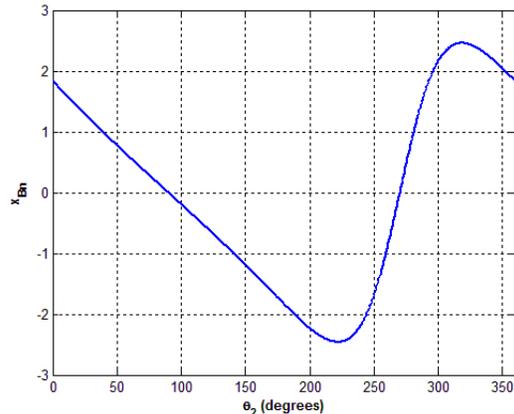


Fig.8 Mechanism normalized output displacement.

### VIII. CONCLUSION

- The synthesis of a 6 bar – 3 sliders planar mechanism was investigated using the nomogram-based synthesis technique invented by the author.
- The nomogram consisted of the four basic functions of any planar mechanism: time ratio, stroke, minimum transmission angle and maximum transmission angle.
- It was possible to synthesize the 6 bar – 3 sliders mechanism for any desired time ratio between 1.6 and 3.
- It was possible to synthesize the 6 bar – 3 sliders mechanism for any desired normalized stroke between 3 and 8.
- The nomogram revealed the minimum and maximum transmission angle during the mechanism operation for cross-check over the performance of the synthesized mechanism.
- A case study was given for a desired time ratio of 2.7 and a desired normalized stroke of 5.
- The normalized dimensions of the mechanism were assigned and the key functions of the mechanism were evaluated analytically.
- The maximum error between the nomogram values and the analytical values was 1.64 %.
- The new technique has proven to be efficient, accurate and does not need any sophisticated mathematical tools.

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