

Synthesis of Planar Mechanisms, Part X: Six Bar-Three Sliders Mechanism (Design II)

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Abstract— This paper presents the optimal synthesis of a six bar – three sliders planar mechanism for motion generation with maximum time ratio. Five functional constraints are used to control the kinematical performance of the mechanism. The MATLAB optimization toolbox is used to optimize the synthesis process. The present mechanism structure is capable of generating an output translational motion with relatively large stroke up to a normalized stroke of 4. It is also capable of generating a time ratio up to 4.1. Transmission angle during a full crank revolution is within the recommended range.

Index Terms— 6 bar-3 sliders mechanism (design II), mechanism synthesis, optimal mechanism dimensions, motion generation, maximum time ratio.

I. INTRODUCTION

Six bar mechanisms are used to provide large time ratio. The mechanism structure presented in the present work is capable of generation motion of an output slider having large time ratio and relatively large stroke.

Hongying, Zhixing, Dewei and Junsheng (2003) presented a numerical comparison method of planar 6-bar dwell mechanism synthesis. They could synthesize the 6-bar mechanism for dwell angle from 25 to 140 degrees and rocker angle from 10 to 160 degrees [1]. Shiakolas, Koladiya and Kebrle (2005) presented a methodology combining differential evolution, evolutionary optimization and geometric centroid positions technique for mechanism synthesis. They applied their methodology to the synthesis of 6-bar linkages for dwell and dual dwell mechanisms with prescribed times transmission angle constraints [2]. Soh, Gracia and McCarthy (2006) obtained four real designs for Watt I system. They showed that the analysis of Watt I six-bar linkage can have four assemblies providing the designer with an opportunity for successful designs when a four-bar linkage is not satisfactory [3].

Kinzel, Schmiedeler and Pennock (2007) extended geometric constraint programming to function generation problems involving large number of finitely separated precision points and complex mechanisms. They presented examples of function generation with a Stephenson III six bar linkage [4]. Chung and Huang (2011) derived the rotation curves of Stephenson III six bar linkage. They concluded that the rotation curve is at most of sixteen degree [5]. Bulatovic and Dordevic (2012) presented the optimal dimensional

synthesis of a six bar linkage with rotational constraints in which a point on the second dyad generates the desired path. The path was generated by a large number of precision points [6].

Shivdas, Bansode, Aulkarni, Parlikar and Ghodekar (2013) evolved the comparison of the four bar mechanism and the six bar mechanism for the application of articulation of a missile container avoiding the use of external crane. They manufactured and realized the six bar mechanism and carried out functional tests [7]. Buskiewicz (2014) proposed a method for solving the problem of a feeder carrying a load between two points. The feeder was assumed to be a 1DOF system of six links connected by R joints. His method enabled decreasing the number of design parameters describing dimensions, orientation and position of a path generator [8]. Hassaan (2014) formulated the synthesis problem of a single dwell six bar linkage. He could synthesize the mechanism for a single dwell during 60 degrees of crank rotation with maximum error less than 0.23 %. He maintained the transmission angle of both four bar part and the coupler-output slider within the recommended range [9].

Tso and Wang (2015) used a computer-aided design approach to study the role of ternary link in Stephenson III six bar mechanical press. They showed that changing the shape and dimensions of the ternary link can get various punch motions [10]. Hassaan (2015) investigated the synthesis of six bar – three sliders mechanism for motion generation having maximum time ratio and an assigned stroke. He assigned the optimal dimensions of the mechanism and kept the transmission angle within the recommended range [11].

II. MECHANISM

The planar 6-bar mechanism under study is shown in Fig.1. It is presented as an problem in Ballany's book 'Theory of Machines' [12]. A line diagram of the mechanism is given in Fig.2.

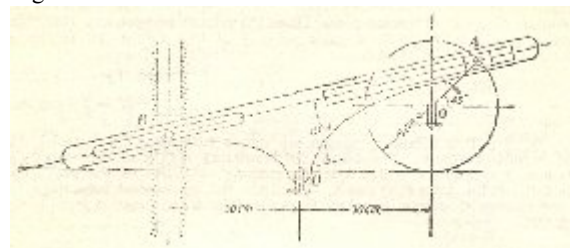


Fig.1 The 6-bar 3-sliders planar mechanism [12].

A line diagram of design II of the 6 bar-three sliders mechanism is shown in Fig.2 to facilitate the kinematical analysis of the mechanism.

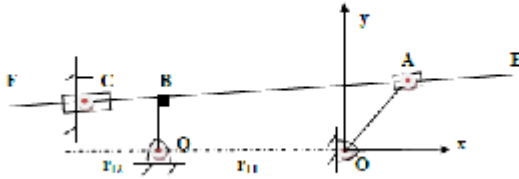


Fig.2 Line diagram of the mechanism.

Its crank OA rotates fully and joined to a slider at A with a R-joint. Slider 3 drives an oscillating L-shaped lever QBEF. Two other sliders are pin-jointed at C on one of them slides over the oscillating lever and the other slides over a vertical guide (the output slider). The mechanism has a unit degree of freedom and its output displacement is joint C vertical coordinate (y_C). The dimensions of the mechanisms having constant length are the crank length r_2 , frame length r_{11} , r_{12} and the oscillating lever dimension r_3 .

III. MECHANISM PERFORMANCE PARAMETERS

The performance parameters of the mechanism are: mechanism time ratio, mechanism stroke, minimum transmission angle and maximum transmission angle. To derive those functional parameters, the mechanism is drawn in its two limiting positions as shown in Fig.3.

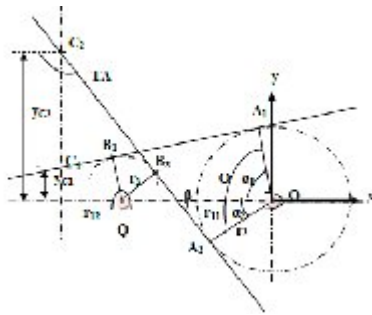


Fig.3 Limiting positions of the mechanism.

The limiting positions of the output slider are C_1 and C_2 corresponding to the crank positions A_1 and A_2 respectively in Fig.3. The mechanism stroke is S (C_1C_2). The crank angle corresponding to the return stroke is \square . The transmission angle is TA measured from the perpendicular line on the line of motion of the output slider at C to the oscillating lever.

The mathematical models of the functional parameters of the 6 bar – 3 sliders mechanism in Fig.2 are as follows:

Time ratio, TR :

The time ratio of a mechanism is defined as the ratio between the time of the forward stroke to the time of the return stroke. In terms of the crank angle \square shown in Fig.3, it is

defined for a constant speed crank by :

$$TR = (360 - \square) / \square \quad (1)$$

Where:

$$\square = \alpha_1 + \alpha_2 \quad (2)$$

α_1 and α_2 are the two angles made by the crank with the negative x-direction as shown in Fig.3. Using the first limiting position of the mechanism as in Fig.3 and drawing a line from Q perpendicular to OA_1 gives α_1 as:

$$\alpha_1 = \cos^{-1} \{ (r_2 - r_3) / r_{11} \} \quad (3)$$

Using normalized dimensions by referring all the dimensions to the crank length r_2 . The normalized lengths of the frame and the oscillating lever become:

$$\begin{aligned} r_{11n} &= r_{11}/r_2 \\ r_{12n} &= r_{12}/r_2 \\ r_{3n} &= r_3/r_2 \end{aligned} \quad (4)$$

Combining Eqs.3 and 4 gives the angle α_1 as:

$$\alpha_1 = \cos^{-1} \{ (1 - r_{3n}) / r_{11n} \} \quad (5)$$

The other limiting position angle α_2 is obtained as follows:

- Let the oscillating lever QA_2B_2 intersect the x-axis in Q' with $QQ' = r_{11}'$.
- From the two triangles OA_2Q' and QB_2Q' , we get:

$$r_{11}' = r_3 r_{11} / (r_2 + r_3) \quad (6)$$

- Using the normalized parameters, Eq.6 becomes:

$$r_{11n}' = r_{3n} r_{11n} / (1 + r_{3n}) \quad (7)$$

- Now, from triangle $OQ'A_2$, α_2 is given by:

$$\alpha_2 = \cos^{-1} \{ r_2 / (r_{11} - r_{11}') \} \quad (8)$$

- Using the normalized dimensions, α_2 becomes:

$$\alpha_2 = \cos^{-1} \{ 1 / (r_{11n} - r_{11n}') \} \quad (9)$$

where r_{11n}' is given by Eq.6.

Stroke, S :

The mechanism stroke S using the geometrical relations of the mechanism limiting positions in Fig.3 is given by:

$$S = y_{C2} - y_{C1} \quad (10)$$

Where:

From the first limiting position, y_{C1} is given by:

$$y_{C1} = [r_3 - r_{12} \sin(90 - \alpha_1)] / \cos(90 - \alpha_1) \quad (11)$$

Using the normalized dimensions, Eq.11 gives the normalized slider position y_{C1n} as:

$$y_{C1n} = [r_{3n} - r_{12n} \sin(90 - \alpha_1)] / \cos(90 - \alpha_1) \quad (12)$$

From the second limiting position, y_{C2} is given by:

$$y_{C2} = (r_{12} + r_{11}') \tan \beta \quad (13)$$

where the angle β is related to α_2 as:

$$\beta = 90 - \alpha_2 \quad (14)$$

Using the normalized dimensions, Eq.13 gives the normalized slider position y_{C2n} as:

$$y_{C2n} = (r_{12n} + r_{11n}') \tan \beta \quad (15)$$

Now, the normalized stroke S_n is given using Eqs.12 and 15 by:

$$S_n = y_{C2n} - y_{C1n} \quad (16)$$

Minimum and maximum transmission angle, TA_{min} and TA_{max} :

Using the geometry of Fig.3, the minimum transmission angle, TA_{min} is given by:

$$TA_{\min} = 180 - \alpha_1 \quad \text{degrees} \quad (17)$$

And the maximum transmission angle, TA_{\max} is given by:

$$TA_{\max} = 180 - \beta \quad \text{degrees} \quad (18)$$

III. OPTIMAL SYNTHESIS OF THE MECHANISM

1. Objective function: The mechanism is to be synthesized for maximum time ratio. That is the time ratio given by Eq.1 is to be maximized.
2. Functional constraints: Functional constraints are set in the optimization technique to control the performance of the mechanism during operation.

Five functional constraints are used to control the performance of the optimally synthesized mechanism:

(i) Normalized stroke constraint:

$$S_n \leq \lambda \quad (19)$$

Where:

λ is a desired value for the normalized stroke.

(ii) Minimum transmission angle constraint:

$$TA_{\min} \geq 45 \quad \text{degrees} \quad (20)$$

(iii) Maximum transmission angle constraint:

$$TA_{\max} \leq 135 \quad \text{degrees} \quad (21)$$

(iv) Fixed joint Q location constraint:

$$r_{11} > r_2$$

In a normalized form:

$$2_{2n} < r_{11n} \quad (22)$$

(v) Oscillating lever constraint:

$$r_3 < r_2$$

In a normalized form:

$$r_{3n} < 1 \quad (23)$$

3. Synthesis dimensions: The mechanism normalized dimensions used in the synthesis process are r_{11n} , r_{12n} and r_{3n} .

4. Synthesis dimensions constraints: The constraints imposed on the synthesis normalized dimensions are as follows:

$$\text{For } r_{11n}: 1.1 \leq r_{11n} \leq 10 \quad (24)$$

$$\text{For } r_{12n}: 1 \leq r_{12n} \leq 10 \quad (25)$$

$$\text{For } r_{3n}: 0.2 \leq r_{3n} \leq 10 \quad (26)$$

5. Optimization procedure: The objective function in Eq.1 is maximized subjected to the functional constraints of Eqs.19 through 26 and the dimensions constraints of Eqs.24, 25 and 26. The MATLAB toolbox is used for this purpose through its command 'fmincon' [13].

6. Optimal mechanism synthesis results: The application of the procedure outlined in this research paper resulted in a very promising features for the 6 bar – 3 sliders mechanism under study. The results are given in Table 1 against the desired stroke of the mechanism.

TABLE I: OPTIMAL MECHANISM DIMENSIONS AND PERFORMANCE FUNCTIONS

λ	r_{11n}	r_{12n}	r_{3n}	TR	S_n	TA_{\min} (deg.)	TA_{\max} (deg.)
1.50	1.8669	1	0.32	2.1677	1.50	111.355	135.000
1.75	1.6146	1	0.20	2.5160	1.75	119.702	131.993
2.00	1.5186	1	0.20	2.7495	2.00	121.788	127.799
2.25	1.4505	1	0.20	2.9688	2.25	123.471	124.179
2.50	1.4008	1	0.20	3.1746	2.50	121.060	124.826
2.75	1.3638	1	0.20	3.3660	2.75	118.370	125.916
3.00	1.3356	1	0.20	3.5426	3.00	116.044	126.796
3.25	1.3138	1	0.20	3.7049	3.25	114.026	127.540
3.50	1.2967	1	0.20	3.8536	3.50	112.266	128.094
3.75	1.3830	1	0.20	3.9896	3.75	110.724	128.571
4.00	1.2719	1	0.20	4.1140	4.00	109.365	128.973

7. The optimal dimensions of the 6 bar – 3 sliders planar mechanism is shown graphically in Fig.4 against its desired normalized stroke from 1.5 to 4.

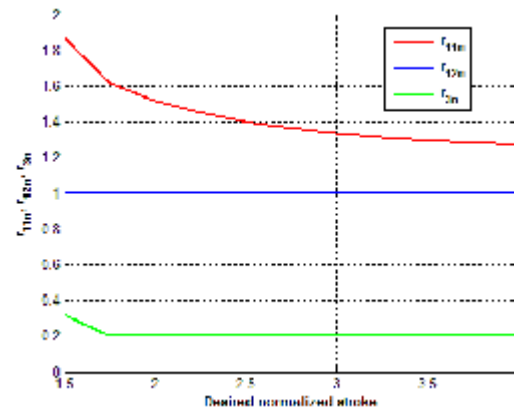


Fig.4. Optimal dimensions of the mechanism against its desired stroke.

8. The optimal time ratio and normalized stroke of the mechanism against its desired normalized stroke are shown in Fig.5.

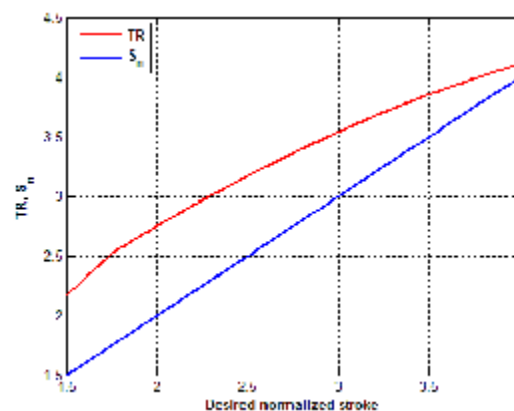


Fig.5 Optimal time ratio and normalized stroke of the mechanism.

9. The optimal minimum and maximum transmission angles of the mechanism against the desired normalized stroke are shown in Fig.6. They are within the recommended range of 45 to 135 degrees [14].

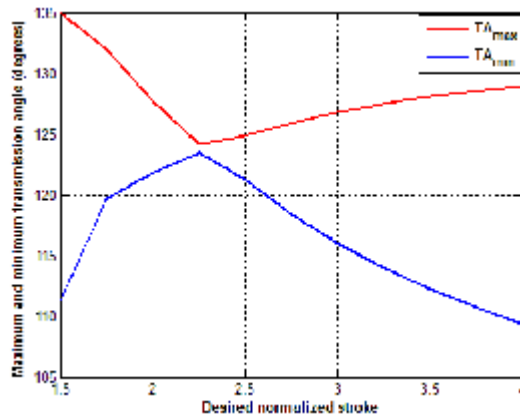


Fig.6 Optimal minimum and maximum transmission angles of the mechanism.

IV. CONCLUSION

- The synthesis of a new structure of a 6 bar – 3 sliders planar mechanism was investigated.
- The maximum time ratio of the mechanism was used as an objective function.
- The performance of the mechanism was controlled through five functional constraints.
- MATLAB optimization toolbox was used to assign the optimal dimensions of the mechanism.
- Normalized dimensions were used in the mechanism synthesis.
- Desired normalized stroke between 1.5 and 4 was assigned.
- The optimal mechanism dimensions were defined against the desired normalized stroke.
- It was possible to achieve a time ratio of the mechanism up to 4.1.
- The transmission angle for the normalized stroke range was between 109.4 and 135 degrees.
- The proposed procedure was completely successful, since the desired stroke was obtained and the transmission angle of the mechanism was within the recommended range for successful mechanism synthesis.

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